Fermi Questions

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Solutions for Fermi Questions, September 2013

Question 1: Losing weight by breathing

How much weight (or mass) do we each lose overnight by breathing as we sleep?
(Thanks to Will Williams of Smith College for suggesting the question.)

Answer: We lose weight as we breathe because we inhale O_2 and exhale CO_2. Therefore we need to estimate how much we breathe every night. We can do this from either volume or energy considerations. To estimate using volumes, we need to estimate the volume per breath, the number of breaths, and the fraction of oxygen converted to CO_2 in each breath. Each breath is about 1 L (more than 0.1 L [a few ounces] and less than 10 L [2.5 gal]). At night we breathe about 10 times per minute (more than 1 and less than 100). The air we breathe is about 20% oxygen, but we only convert about 10% of that to CO_2 (since our exhalations contain enough oxygen for CPR to work). Therefore, in one night we exhale

$$V = (8\text{ hr})(60\text{ min/hr})(10\text{ br/min})(1\text{ L/br})$$
$$= 5\times10^3\text{ L} = 5\text{ m}^3$$

which has a mass of carbon

$$m_C = \frac{V}{20\text{ L/mole}}(0.2)(0.1)(12\text{ g/mole})$$
$$= 60\text{ g} = 0.06\text{ kg}.$$

Five cubic meters of air has a mass of about 5 kg. However, only 1 kg of that is oxygen and we only convert 0.1 kg of oxygen to about 0.15 kg of CO_2.

Now let's consider energy. We each consume about 2500 Cal = 10^7 J of chemical energy daily, all of which needs to be oxidized. At 1.5 eV per chemical reaction (we can estimate that either from the 15-eV binding energy of hydrogen or from the 1.5 V potential of the standard AA, AAA, C or D battery), this means we need

$$N_{O_2} = \frac{10^7\text{ J/day}}{(1.5\text{ eV/O}_2)(1.6\times10^{-19}\text{ J/eV})(6\times10^{23}\text{ \#/mole})}$$
$$= 60\text{ moles/day O}_2,$$

which increases in mass as it becomes CO_2 by

$$\Delta m = m_c = (60\text{ moles/day})(12\text{ g/mole})$$
$$= 7\times10^2\text{ g/day} = 0.7\text{ kg/day}.$$

Therefore overnight we exhale one-third of this, or 0.2 kg of carbon.

These two estimates differ by a factor of three. This is within the uncertainty of the volumetric estimate since the volume per breath and the breaths per minute estimates both have uncertainties of about a factor of three. (We can estimate the uncertainty of a given estimate by taking the square root of the ratio of the estimate and its lower bound. In other words, if our lower and upper bounds are 1 and 100, our estimate is 10 with an uncertainty of $\sqrt{10} = 3$. This assumes that our upper and lower bounds are each two standard deviations from the mean.)

In either case, this is a measurable number. With a good enough scale (and enough pre- and post-sleeping measurements) we should be able to determine this number experimentally.

Question 2: Too many ads

How many television advertisements has a typical American 18-year-old seen? How much time has been spent watching ads?

Answer: In order to estimate the number of advertisements seen, we need to estimate the number of hours per day spent watching television and the number of commercials per hour. The average American spends more than one hour and less than 10 hours per day watching television; we will take the geometric mean and estimate three hours per day. In any given half-hour there are about 10 minutes of ads (more than 10% and less than 100%). Each ad is about 0.5 minute, giving 40 ads per hour, or 100 ads per day.

Thus, by age 18, the typical American has seen

$$N_{ads} = (10^2\text{ ads/day})(4\times10^2\text{ days/yr})(20\text{ yr})$$
$$= 10^6\text{ ads}.$$

This is probably an overestimate since they were not watching three hours of TV per day at age 3 (I hope!), some of the time was used for productive activities such as micturition, and some of the commercials were skipped by channel surfing.

Now let's calculate the time:

$$t_{\text{ads}} = \frac{(10^6\ \text{ads})(30\ \text{s/ad})}{10^5\ \text{s/day}} = 300\ \text{days}$$

$$\approx 1\ \text{yr.}$$

That is about 5% of their life. Wow.