Fermi Questions

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Solutions for Fermi Questions, November 2013

Question 1: Melting a car

Sunlight reflecting from London's Walkie Talkie tower was accused of partially melting a parked Jaguar in September. Is this reasonable? (*Thanks to Boris Korsunsky of Weston High School and* The Physics Teacher *for suggesting the question.*)

Walkie Talkie tower in London. (iStock Photo)

Answer: To estimate the effect, we need to estimate the reflective area of the Walkie Talkie tower, the focusing power of the building, the intensity of unfocused sunlight, and the area, mass, and specific heat capacity of the plastic panels on the affected car. The building has more than 10 and fewer than 100 stories. At 3 m per story (about 10 ft), this gives a height estimate of 100 m. The building is between 1 and 10 times taller than it is wide, giving a width of 30 m. Thus the reflective area of the building is $3\times10^3 \text{m}^2$.

Now we need to estimate the focusing power of the building. We can do this in one of two ways. We can estimate that the building focuses more than 0.1% and less than 10% of the incident light on a 1-m^2 area. Alternatively we can estimate that the building focuses in one dimension only so that all the incident light is focused on an area that is 30 m by 3 m (more than 1 m and less than 10 m). In the first case we estimate that the building focuses 30 m^2 of light onto a 1-m^2 area. In the second case we estimate that the building focuses $3\times10^3 \text{m}^2$ of light onto a 10^2-m^2 area. In either case, we get a focusing factor of 30.

The solar flux at Earth orbit is 10^3 W/m^2 and is about half that at the Earth's surface. This gives a focused light intensity of

$$S = (0.5)(10^3 \text{W/m}^2)(30) = 10^4 \text{W/m}^2.$$

Imagine the heat of sunlight on a very sunny day. Now multiply by 30. Where's my parasol?

This is a very large flux, but can it damage a car? Consider a plastic panel on the outside of a car with area A. It will have a thickness $t \approx 3$ mm (more than 1 mm and less than 1 cm), a density $\rho \approx \rho_{\text{water}} = 10^3 \text{kg/m}^3$, and a mass $m = \rho A t$. It will absorb power $P = SA$ so that the power density (power absorbed per mass) will be

$$\begin{aligned} p &= P/m = S/(\rho t) \\ &= \frac{10^4 \text{ W/m}^2}{(10^3 \text{ kg/m}^3)(3\times10^{-3} \text{m})} \\ &= 3\times10^3 \text{ W/kg} = 3 \text{ W/g} \end{aligned}$$

The specific heat of water, $c = 4$ $J/(\text{g.K})$, is about an order of magnitude greater than that of other materials. Thus, the temperature of the plastic will increase at a rate

$$\begin{aligned} \dot{T} &= p/c \\ &= \frac{3 \text{ W/g}}{0.4 \text{ J/(g}\cdot\text{K)}} \\ &= 10 \text{ K/s} \end{aligned}$$

Uh-oh. At this rate the plastic will melt in only 10 to 20 seconds. This means that even if we include the reflectivity of plastic and heat conduction to other parts of the car, the plastic is very likely to melt.

This is why concave buildings need careful optical design. On the other hand, concentrated sunlight would be quite welcome in the next few months.

Question 2: Google cars

How many cars does Google need to take its street view pictures in the United States?

Answer: In order to estimate this, we need to estimate the number of street miles, the average car speed, and the picture update frequency. Let's start with the number of street miles. The majority of Americans live in detached houses with a street frontage between 10 and 100 m (30 and 300 ft), giving 30 m per house. We should divide this by a factor of about two to account for household size and conveniently multiply by about the same factor to account for non-residential roads. Multiplying by the population of the U.S., this gives a total

$$d = (30 \text{ m/person})(3\times10^8 \text{ persons})$$
$$= 10^{10} \text{ m} = 10^7 \text{ km}$$

of roads in the United States. Making the dreaded reality check, we find that there are 7×10^6 km of roads in the U.S. (at least according to Wikipedia, the modern arbiter of all knowledge).

The average street view camera car will travel faster than 10 and slower than 100 mph, giving an average speed of 30 mph or 50 km/hr. (I'm sorry. Despite teaching physics for decades, I still think in mph, not km/hr or m/s.) If the car is driven continuously during standard working hours, then it will be driven 2×10^3 hr/yr (8 hr/day$\times$5 day/wk $\times$50 wk/yr) and travel a distance of

$$d_{car} = (50 \text{ km/hr})(2\times10^3 \text{ hr/yr})$$
$$= 10^5 \text{ km/yr.}$$

Now the number of cars depends solely on how often Google chooses to update the street view pictures. The update period will be longer than one year and shorter than 10 years, giving a period of three years. A three-year update period will require

$$N_{cars} = \frac{10^7 \text{ km}}{(10^5 \text{ km/yr})(3 \text{ yr})} .$$
$$= 30$$

Is this a lot or a little? Let's also estimate the cost of taking all those pictures. The labor costs of driving each car will be about $\$3\times10^4$ per year (more than $\$10^4$ and less than $\$10^5$). The purchase, maintenance, and gasoline costs of driving will be about \$0.5 per km or $\$5\times10^4$ per car per year. Thus, the total costs of taking all the pictures will be about

$$C = (30 \text{ cars})(8\times10^4 \$ \,/\, \text{car})$$
$$= \$2\times10^6$$

not including the costs of incorporating them into Google street view.

Is this a little or a lot? Compared to the income of a typical person, it is definitely a lot. Compared to Google's annual revenues of about \$30 billion (more than \$1 billion and less than \$1 trillion), it is almost negligible.