Fermi Questions

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Question 1: Flying south

How much energy do birds need to fly south for the winter? How much fat would they need to make the journey (as a percentage of their body weight) if they make the entire trip without eating? Alternatively, how much food do they need to consume en route?

Answer: In order to estimate this, we need to estimate the distance that the birds travel, their flying efficiency, and the caloric value of their food (or their fat). Let's start with the distance traveled. The minimum migration distance is about 10^3 km and the maximum is the distance from the North Pole to the South Pole or 2×10^4 km.

The flying efficiency will depend on the size and weight of the bird, the shape and size of their wings, how often they flap their wings, and many other factors. Fortunately, we can ignore all of these details. When a bird is gliding, it will travel a certain horizontal distance for each meter of height lost. The ratio of horizontal distance traveled to height lost is the glide ratio. We can confidently estimate that the avian glide ratio is more than 1 and less than 100 so we will estimate 10. We will further estimate that the mechanical efficiency of flight (to regain the lost altitude) is 1/3 (more than 1/10 and less than 1). Therefore, an ideal 1-kg bird will lose 1 m of altitude every 10 m and expend 30 J of chemical energy to regain the lost 10 J of potential energy. (Most birds mass much less than 1 kg. The arctic tern, which migrates from pole to pole each year, has a mass of only about 100 g.)

Extrapolating this to a 10^3 km migration, at 3 J/m our ideal 1-kg bird will need 3×10^6 J of food energy (or about 10^3 kcal). Our bird will need 20 times as much food for a 2×10^4 km migration, or about 6×10^7 J or 2×10^4 kcal.

Let's pause for a sanity check. Our 1-kg bird expends about 1 kcal (food calorie) per km. Scaling linearly, a 100-kg human would need 100 kcal per km. This is not crazy.

Now we need to estimate the caloric value of food or fat. We might remember that sugar has 16 kcal/tsp (3 kcal/g = 10^4 J/g = 10^7 J/kg) and that fat has three times the energy density of sugar (3×10^7 J/kg). We might remember (or estimate) the energy density of gasoline as $e = 3\times10^7$ J/kg and estimate that fat has almost as much energy density as gasoline.

This means that our 1-kg ideal bird needs to start with 0.1 kg of fat to fly 10^3 km. Note that this does not include the demands of its basic metabolism. If we double this to account for basic metabolism, then our bird needs to start with about 20% fat to fly 10^3 km. If it eats typical high-protein foods (which will be much more similar in caloric content to sugar than fat), it will need to consume 30% to 60% of its body weight in food to make the journey.

To make a 2×10^4 km journey, our 1-kg bird will need to start with 2 kg of fat. Oops, now it is no longer a 1-kg bird but a 3-kg bird and will need commensurately more fuel. Our poor bird will definitely need to stop to eat en route. It will need to consume between 6 and 10 times its body weight in high-energy foods to make this journey.

That's a lot of food!

Question 2: Election words

How many words were spoken in public by all of the presidential candidates during the 2012 election season?

Answer: In order to estimate this, we need to estimate the length of the election season, the number of candidates speaking, and the number of words they speak in public each day. While it feels like the election season lasts forever, it is really only about a year. Similarly, despite the fact that each of the primary debates appeared to have 17.3 candidates on stage, the average number of candidates during the primary season was only about five or six and there were only two candidates during the general election.

Therefore we will estimate that there are five candidates for the 200-day primary season and two candidates for the 200-day general election season for a total of 10^3 candidate-days.

Now we need to estimate the number of words spoken in public per candidate per day. Let's break this down into the number of hours spent speaking and the number of words per hour. We can estimate the speaking time in two ways. First we can bound a candidate's public speaking time at more than one and less than 10 hours per day. Taking the geometric mean, we estimate that each candidate spends three hours per day addressing the public. Alternatively, we know that candidates divide their waking time between fundraising, traveling, organizing, and speaking publicly. Assuming that they sleep eight hours and divide the remaining time equally, this gives four hours per day for public speaking. Some candidates might even spend some of their time listening. If so, then they will spend three hours per day in public speaking.

Speaking speed can be estimated a few ways. We can listen to a speech and measure it, we can take our reading speed and divide by a factor of several, we can look at the transcript of the speech, or we can bound it. Let's bound it. People speak more than 10 words per minute and less than 10^3. Taking the geometric mean, we get 10^2 words per minute. This makes sense because it is several times slower than typical reading speeds and it is faster than all but the fastest typing speeds.

Now the number of words uttered in public during the campaign can be calculated as

$$\begin{aligned} N &= (10^3 \text{ candidate-day})(3 \text{ hr/day})(60 \text{ min/hr}) \\ &\quad (10^2 \text{ words/min}) \\ &= 2\times 10^7 \text{ words.} \end{aligned}$$

That is 20 million words. At 600 words per page, that is enough to fill several hundred books. The reader can decide for him or herself how much all those words are worth.