Fermi Questions

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Solutions for Fermi Questions, May 2014

Question 1: Heat-powered flashlight

One of the 2013 Google Science Fair prizes was awarded for a flashlight that uses Peltier tiles to harness the temperature difference between your hand and the flashlight. How much power could this generate? How does this compare to the power generated by shaking an induction flashlight? Will this change the world (as claimed by Yahoo News)? (*Thanks to Boris Korsunsky of Weston High School, Weston, MA for suggesting the problem.)*

Answer: The flashlight body uses Peltier tiles between your hands and a hollow aluminum tube to convert some of the heat that flows from your hands to the tube to electrical energy (see http://googleblog.blogspot.com/2013/09/and-winner-of-2013-google-science-fair.html and http://www.cbc.ca/news/technology/body-heat-powered-flashlight-takes-teen-to-google-science-fair-1.1317745). The heat lost by your hands will be replaced by convective heating from blood circulation. Therefore, the limit on the steady-state power generated will be the heat flow from the blood.

In order to estimate the power generated, we need to estimate the heat flow from the blood and the temperature difference between your hands and the air. The body dissipates 100 W of heat. One hand will dissipate much less than that. We will estimate the heat flow to one hand as between 1% and 10% of 100 W, or about 3 W.

The maximum temperature difference is limited by our cold tolerance. If the temperature is much below 50°F (10°C) most people wear gloves. While our core temperature is remarkably constant at 37°C, the temperature of our hands varies significantly (as you may have noticed when a "friend" placed their freezing hands on the back of your neck). Let's assume a maximum comfortable temperature difference of 20°C.

The maximum thermodynamic efficiency is

$$\varepsilon_{\text{carnot}} = 1 - \frac{T_C}{T_H}$$
$$= 1 - \frac{280\ \text{K}}{300\ \text{K}} = 6 \times 10^{-2}.$$

However, this only applies for quasistatic (really, really, reallllllllllllly slow) processes. The maximum thermodynamic efficiency for heat engines that maximize power output is

$$\varepsilon_{\text{MaxPower}} = 1 - \sqrt{\frac{T_C}{T_H}}$$
$$= 1 - \sqrt{\frac{280\ \text{K}}{300\ \text{K}}} = 3 \times 10^{-2}.$$

(This factor of two is irrelevant here. I just prefer to use the maximum power efficiency equation because it is much more realistic. Sorry, even a professional guesstimator can be pedantic at times.)

Thus, the maximum sustained power output will be

$$P = \varepsilon Q$$
$$= (3\times10^{-2})(3\ \text{W}) = 0.1\ \text{W}.$$

This is not much at all. It is a factor of several less than the power generated by shaking an induction flashlight, $P = (20\ \text{J})/(30\ \text{s}) = 0.6\ \text{W}$ (Fermi Column, *TPT*, March 2012). But it's a beautiful concept that earned a well-deserved award for Ann Makosinski of Victoria, BC.

Unfortunately, the flashlight only works when it's cold out. So it's not terribly surprising that it was invented by a Canadian.

Question 2: Carpeting the world

What is the total area of all the carpet in the world?

Answer: Let's start by estimating the carpet in one country and then scaling to the world. The vast majority of carpet is indoors, so we'll estimate the total indoor area and the proportion that is carpeted. I live in the United States so let's start there. The total indoor area is between 10 and 1000 m^2 per person, so we'll estimate 10^2 m^2. (I must confess that I estimated the answer in square feet and then divided by 10 to get square meters. Even after decades of teaching physics, I still think in U.S. Customary units.)

The proportion of this area that is carpeted will be more than 10% and less than 100%, so we will estimate 30%. (Even if you increase the lower bound to 50%, it only changes the estimate by a factor of two.)

The relative areas of carpet in the United States and the entire world will scale by either population or income (GDP). This means that the U.S. has between 5% (population scaling) and 25% (income scaling) of the world's carpet, with a geometric mean of about 10%.

Thus, the total area of carpet in the world is

$$A = (3\times10^{8}\ \mathrm{Am})(10^{2}\ \mathrm{m}^{2}/\mathrm{Am})(0.3)(10)$$
$$= 10^{11}\ \mathrm{m}^{2} = 10^{5}\ \mathrm{km}^{2}.$$

That equals the surface area of Virginia, Hokkaido, or Portugal, about 1% of the surface area of the United States, or about 0.02% of the surface area of the globe.

If only so much of it wasn't shag carpet.