Fermi Questions

Larry Weinstein, Column Editor
Old Dominion University, Norfolk, VA 23529; weinstein@odu.edu.

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Question 1: Horse exhaust

If we still rode horses for transportation our pollution problems would be quite different. How much exhaust does one horse produce annually? How much if we replaced all of our cars and trucks with horses?

Answer: Automobiles, for all their faults, produce a gaseous exhaust composed almost entirely of water vapor and carbon dioxide. Horse exhaust is less benign. To estimate the amount of horse exhaust we will scale up from human exhaust.

Humans masses typically range from 50 to 100 kg. Horses are about 10 times larger (more than 1 and less than 100). Food consumption for mammals (which maintain a constant internal temperature) should be proportional to surface area, which is proportional to the typical length squared, which is proportional to mass or volume to the 2/3:

$$A \propto l^2 \propto V^{2/3} \propto m^{2/3}.$$

Since horses have 10 times the mass of humans, they should consume about five times more food and hence produce five times more solid and liquid waste. (This is probably over-precise. Linear scaling would be much simpler and about as accurate for our purposes.)

Humans consume about 1 kg of food per day (more than 100 g and less than 10 kg) and about 2 kg of liquids (if you follow the discredited eight cups of water per day rule). This means that a horse consumes 20 kg of food and water daily, and, by conservation of mass, produces 20 kg of waste. In one year, that horse will produce 7000 kg of waste, which would occupy 7 m^3.

An automobile burning 6 L of gasoline per day (500 gal/yr) will also produce about 20 kg of exhaust per day (since burning $CH_2 \rightarrow CO_2 + H_2O$ increases the molecular weight by a factor of about three). However, if the gasoline is properly burned and the exhaust is properly treated, then this exhaust does not smell and is locally benign.

There are 2×10^8 cars in the United States (slightly less than one per person). If we replaced all of them with horses, they would produce 10^9 tons of horse exhaust yearly. This is several times larger than all of the municipal solid waste we generate.

The entire idea of replacing cars with horses is "Horse s***!"

Question 2: Feeding transportation

How much more land would we need for crops if we still used horses for transportation? Assume that we replace every car with a horse.

Answer: We "feed" cars with "food" we pump from underground. We feed horses with food we grow above ground. If we still used horses for transportation rather than automobiles, we would need to grow a lot more food on a lot more farmland.

As we determined in the previous question, a horse consumes about five times as many calories as a human. Horses have less efficient digestive systems than humans (solid horse exhaust contains a lot of undigested grains) but humans eat a lot more meat (each meat calorie requires two to 10 plant calories to produce). Thus the 2×10^8 horses will consume about twice as many primary food calories as the 3×10^8 humans.

Now let's estimate the land area currently devoted to food production. We can estimate this in a few ways. It is about 1 acre per person (more than 0.1 and less than 10). If you prefer metric measurements, it is about 1 hectare per person (more than 0.1 and less than 10). The second estimate is 2.5 times the first, which gives a measure of the uncertainties involved. This gives a total farm area of

$$A = (3\times10^8 \text{ ha})(10^{-2} \text{ ha/km}^2)$$
$$= 3\times10^6 \text{ km}^2.$$

Alternatively, we could estimate that farms are about 10% (more than 1% and less than 100%) of the land area of the contiguous U.S., neglecting Alaska because it is too cold and Hawaii because it is too small. The width of the contiguous U.S. is three time zones or 1/8 of the circumference of the globe and the height is less than half of the width, giving a total farm area of

$$A = (0.1)(5\times10^3 \text{ km})(2\times10^3 \text{ km})$$
$$= 10^6 \text{ km}^2.$$

Hmmm. If we had used one acre per person rather than one hectare, the two estimates would agree surprisingly well.

On the third hand, we could estimate the land needed for food production by assuming that 1% of the solar energy incident on the land is converted to food calories. That estimate is left as an exercise for the reader.

The actual total farmland, according to the U.S. Department of Agriculture, is 10^9 acres (4×10^8 ha or 4×10^6 km^2). This is within a factor of a few of our estimates (although it is 40%, rather than merely 10%) of the U.S. land area.

Thus, in order to feed 2×10^8 horses, we would need an additional 8×10^6 km^2 of farmland, for a total of 120% of the surface area of the United States. One reason for the reforestation of the East Coast of the U.S. in the 20th century was the diminished demand for horse fodder with the introduction of the automobile.

I think I'll keep my car for now.