Fermi Questions

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Solutions for Fermi Questions, March 2012

Question 1: Shaking flashlights

Induction flashlights have a hollow clear plastic tube containing a coil of wire in the middle, a moveable magnet, and fixed repulsor magnets at the two ends. When you shake the flashlight, the magnet goes back and forth through the coil generating electrical energy, which is then stored in a capacitor and later used to light the LED. How much electrical energy can you generate by shaking the flashlight?

Answer: In order to calculate the electrical energy generated by shaking the flashlight, we could use the precise magnetic field configuration due to the magnet, the number of coils of the wire, the resistance of the circuit, and … Oh, never mind. Let's start over.

Forget the details of the circuit; let's consider the shaking. We shake the flashlight so that the magnet will oscillate back and forth through the coil. During each cycle (or shake), some fraction of the magnet's kinetic energy will be transferred to electrical energy. This fraction must be less than 100% and more than 10%, so we will estimate 30%. This means that we just need to estimate the shaking frequency and the magnet speed and mass. Fortunately, they are related.

When we shake the flashlight, the frequency is about 3 Hz (more than 1 Hz and less than 10 Hz), and the amplitude is about half the length of the flashlight or about 10 cm. Therefore, the peak velocity of the magnet passing through the coil is

$$v = 2\pi A f = 6(10\text{ cm})(3\text{ Hz}) = 2\text{ m/s}.$$

The mass of the magnet is significantly less than the mass of the flashlight. Let's estimate that $m = 0.1$ kg (more than 0.01 kg and less than 1 kg). This means that the maximum kinetic energy of the magnet is

$$KE = 0.5mv^2 = (0.5)(0.3\text{ kg})(2\text{ m/s})^2 = 0.2\text{ J}.$$

The electrical power generated will then equal

$$P_e = (0.1)(KE)f = (0.3)(0.2\text{ J})(3\text{ Hz}) = 0.2\text{ W}.$$

This seems low.

However, we need a reasonable standard to compare it to. The induction flashlight uses a high-capacitance, low-voltage capacitor. These typically have about $C = 1$ F and $V = 6$ V for a total energy storage of

$$E = (1/2)CV^2 = (0.5)(1\text{ F})(6\text{ V})^2 = 20\text{ J}.$$

At 0.2 W, this capacitor can be completely charged in

$$t = E/P = (20\text{ J})/(0.2\text{ W}) = 10^2\text{ s},$$

or 2 minutes. This is slightly longer than the claimed charging time (30 s) for the flashlight.

These flashlights are successful despite the inefficiency of shaking as a charging mechanism. They are successful because of the very low power and high efficiency (0.07 W and 30%) LEDs and because of the development of small 1-F cpacitors.

Question 2: The sky is falling

Heavy things sometimes fall from the sky, including meteorites, satellites, and bullets from celebratory or antiaircraft gunfire. What is the probability that a falling object will hit somebody?

Answer: The probability that a falling object will hit a person depends on the cross section of the person and the areal density of people. For a small, vertically falling object, the cross section of a person is just the area as viewed from above (the pigeon's eye view). This will be about the thickness of your torso times the width of your shoulders, or about

$$A = tw = (0.2\text{ m})(0.6\text{ m}) = 0.1\text{ m}^2/\text{person}.$$

For a large object such as a satellite, the cross section can be as much as 10 or 10^2 m^2. Fortunately, there are many fewer large falling objects.

The areal density of people is just the number of people divided by the area of the Earth or

$$\rho = \frac{7\times10^9\text{ people}}{4\times10^{14}\text{ m}^2} = 2\times10^{-5}\text{ people/m}^2.$$

Thus, the probability that a small object will strike a person as it plummets to Earth is

$$P = \rho A = (0.1\text{ m}^2/\text{person})(2\times10^{-5}\text{ people/m}^2) = 2\times10^{-6}.$$

The probability that a large object will strike a person is 10^2 to 10^3 times greater, or as much as 10^{-3}.

Thus, when a large satellite falls to Earth, the chance that it will hit someone is less than 10^{-3}.

Note that, in aggregate, small objects are more dangerous. In many parts of the world celebrations include celebratory gunfire. This is not a problem if the projectiles are very small (0.22/5.56 caliber) but can be dangerous for larger rounds.

In this case, we need to revise the population density because celebratory gunfire is most often performed in populous areas. If we use a typical urban density of 10^3 people/km^2 (more than 10^2 and less than 10^4) then we have

$$P = \rho A = (0.1 \text{ m}^2/\text{person})(10^{-3} \text{ people/m}^2) = 10^{-4},$$

or one injury for every 10^4 rounds fired. Note that this can be somewhat decreased if most people are indoors and it can be substantially increased if everyone is outside together in the town square.

As expected, casualties from meteorites and falling satellites are quite rare, but casualties from bullets returning to Earth are not unusual.