Fermi Questions

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Question 1: Red to Dead

The level of the Dead Sea has dropped significantly in recent decades due to diversion of the Jordan River water by bordering countries. There is a new international agreement to replace this diverted water with water piped from the Red Sea, about 100 km to the south. How much water flow is needed to stabilize the Dead Sea level? How much power could this generate?

Answer: To estimate this, we need to estimate the surface area of the Dead Sea and the rate at which its level is decreasing. The Dead Sea is about 100 km long (more than 10 and less than 1000 km) and about 30 km wide (the aspect ratio is more than 1 and less than 10), giving an area of 3×10^3 km^2 (or 3×10^9 m^2). Comparing to reality (or at least Wikipedia's version thereof), this estimate is a factor of five too high. On the other hand, the Dead Sea has shrunk a lot in recent years.

The sea level decrease is less certain. Over the last few decades the sea level has decreased more than 1 m (negligible) and less than 1 km, giving an estimate of 30 m. In order to work with round numbers, we'll assume the time period is 30 years so that the sea level decrease rate is 1 m/yr.

The missing volume of water is

$$\begin{aligned}\dot{V} &= A\dot{h} = (3\times10^9\,\text{m}^2)(1\text{ m/yr})\\ &= 3\times10^9\,\text{m}^3/\text{yr}\\ &= 10^2\,\text{m}^3/\text{s}\end{aligned}$$

(using the usual conversion factor that 1 year = $\pi\times10^7$s). That is a manageable amount. We could do that with water flowing through a 10-m^2 pipe at 10 m/s (20 mph). If we used the correct Dead Sea surface area, we would only need 20 m^3/s which is even easier.

The shore of the Dead Sea is the lowest point on Earth. Water piped from the Red Sea to the Dead Sea will thus travel down hill. To estimate the power we could generate, we need to estimate the height differential. This will be more than 100 m and less than 1 km, or about 300 m. Ignoring frictional losses, water flowing through this height difference can provide power

$$\begin{aligned}P &= \dot{m}gh = \dot{V}\rho gh\\ &= (20\text{ m}^3/\text{s})(10^3\,\text{kg/m}^3)(10\text{ m/s}^2)(300\text{ m})\\ &= 6\times10^7\,\text{W},\end{aligned}$$

which is quite respectable (but still much less than the 10^9 W generated by large modern electrical power generators).

Question 2: Dead evaporation

How much water evaporates from the Dead Sea every year?

Answer: To estimate this, we need to estimate the surface area of the Dead Sea, the amount of solar energy it receives, and the latent heat of vaporization of water. We estimated the surface area of the Dead Sea in the previous question as $A = 3\times10^3$ km^2 and then found that in reality it was a mere 600 km^2. Since the Dead Sea has shrunk a lot in recent decades and because we like round numbers, we'll use $A = 10^3$ km^2.

The solar flux at Earth orbit is $S = 10^3$ W/m^2. Since the surface area of the Earth is $4\pi r^2$ but the intercepted radiation only fully illuminates a circle of area πr^2, we'll divide by four to account for the varying angle of incidence of solar radiation during the day and for night. We will neglect clouds. The total solar flux at the Dead Sea is thus

$$\begin{aligned}P &= SA = \frac{1}{4}(10^3\,\text{W/m}^2)(10^3\,\text{km}^2)\\ &= 2\times10^{11}\,\text{W}\quad.\end{aligned}$$

While we could easily look up the latent heat of water, that seems somehow unsporting. We will estimate it from the specific heat of water and from the relative time needed to heat water to the boiling point on the stove and to boil away the water (convert it entirely to vapor and then burn the pot). The specific heat of water is 1 cal/gK (or 1 BTU/lb ºF). Thus it takes 80 cal (or about 300 J) to raise the temperature of 1 g of water from 20 to 100 ºC. When cooking, it takes several times longer to boil away the water than to bring it to a boil. If it takes 10 minutes to bring water to a boil, it will boil away in about an hour (more than 10 minutes and less than three hours). This factor of six gives a latent heat of vaporization

$$L = 6\times(300\ \text{J/g}) = 2\times10^3\ \text{J/g},$$

which is remarkably close to reality.

The efficiency of Dead Sea sunlight for evaporating water (including reflection and other processes) is about 30% (more than 10% and less than 100%). The rate of evaporation is

$$\begin{aligned}\dot{m} &= 0.3\frac{P}{L} = 0.3\times\frac{2\times10^{11}\ \text{W}}{2\times10^{3}\ \text{J/g}}\\ &= 3\times10^{7}\ \text{g/s}\\ &= 3\times10^{4}\ \text{kg/s}\ .\end{aligned}$$

In volume terms, this is

$$\begin{aligned}\dot{V} &= \frac{\dot{m}}{\rho} = \frac{3\times10^{4}\ \text{kg/s}}{10^{3}\ \text{kg/m}^{3}}\\ &= 30\ \text{m}^{3}\text{/s}\ .\end{aligned}$$

This amount is remarkably close to the flow rate needed to maintain the level of the Dead Sea as calculated in the previous problem. This implies that almost all of the fresh water sources of the Dead Sea are now being used for drinking, irrigation, and other purposes.