Fermi Questions

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Question 1: Balloons to lift house

In the movie "Up," the hero flies away by attaching lots of balloons to his house. How many balloons would you need to lift your house? (Thanks to John Adam of Old Dominion University and coauthor of *Guesstimation* for suggesting the question.)

Answer: To answer this we need to estimate the weight of a typical house and the lift of a typical balloon. The typical American house has an area of about 3000 ft^2 (more than 10^3 and less than 10^4 ft^2) or about 300 m^2. This implies a volume of about 10^3 m^3. The structural volume will be more than 1% and less than 25% of the house volume so we will estimate 5%. Since most houses are built of wood, the density will be about 10^3 kg/m^3.The mass of the average American single family home is then

$$m = \rho V = (5\times10^{-2})(10^3\ \mathrm{kg/m^3})(10^3\mathrm{m^3})$$
$$= 5\times10^4\ \mathrm{kg} = 50\ \mathrm{tons.}$$

Air has a density of 1 kg/m^3, so we will need to displace 5×10^4 m^3 of air in order to lift 5×10^4 kg of building. A typical large party balloon has a volume of about 0.1 m^3 (more than 0.01 m^3 [10 L] and less than 1 m^3) so we would need 5×10^5 party balloons. Weather balloons have a volume of about 10 m^3 (more than 1 and less than 10^2) so we would only need 5×10^3 of those. We will need many more balloons than the Macy's Thanksgiving Day parade in New York City!

Now let's check with reality, such as it is. A National Geographic team of scientists, engineers, and balloon pilots used 300 8-ft weather balloons to lift a 1-ton 25-m^2 house. This set a world record for the "largest balloon cluster flight." Our 10-times-larger house, which was not optimized for balloon flight, weighs 50 times more and we estimated that it would require almost 20 times as many weather balloons.

Our estimates are pretty good and I am definitely not planning to take my house flying any time soon.

Question 2: Recycled toilet paper

Friends of mine switched to using recycled toilet paper to "help the environment." However, they revert to using softer toilet paper when guests visit. How much environmental benefit do they accrue from using scratchy toilet paper?

Answer: It's important to use recycled paper products because paper does not grow on trees. Oh. Wait a minute. Never mind. Let's start again.

In order to estimate the benefit of using recycled toilet paper, we will estimate the amount of toilet paper used yearly and the savings from reusing wood pulp. We can estimate the toilet paper usage in several ways. First, we can start with individual pieces of toilet paper. We each use the toilet a few times a day and use more than three and fewer than 30 sheets per use (if we're female), giving a total use of 30 sheets per day. At 1000 sheets per roll, that means we use one roll per person per month. Alternatively, my wife and I change toilet paper rolls more often than once a month and less than once a week, giving a total use of two rolls per month at home. Doubling this to include work, we use two rolls per person per month. On the gripping hand, we purchase large multi-packs of toilet paper once or twice a year, which gives about the same usage estimate.

Now let's estimate the mass of that toilet paper. One roll of toilet paper weighs more than one ounce and less than one pound, so we will estimate 1/4 pound. Thus the 24 toilet paper rolls each that my friends use yearly weigh a total of 6 pounds or 3 kg.

Let's estimate the amount of land needed to grow the wood-fiber for 3 kg of paper. A loblolly pine tree can grow to 60-cm diameter and 10-m tall in about 20 years. Ignoring the mass of the branches, the tree has a mass of

$$m_{\text{tree}} = \frac{1}{3}\pi r^2 h\rho$$
$$= (0.3\ \mathrm{m})^2(10\ \mathrm{m})(10^3\,\mathrm{kg/m^3})$$
$$= 10^3\,\mathrm{kg},$$

or about 50 kg/year. If trees are spaced every 5 m, that gives a mass production density of 2 kg/(yr-m^2). Checking with reality, we find that net biomass productivity in forests ranges from 1 to 2 kg/(yr-m^2) (see "The Physical Environment" by Michael Ritter, online at www4.uwsp.edu/geo/faculty/ritter/geog101/textbook/biogeography/biomass_productivity.html). Thus the fiber for your toilet paper can be grown on about 2 m^2 of land.

Let's compare this to our other agricultural consumption. The contiguous United States is about 3000 miles across (three-time-zone difference from East to West Coast) and about 1000 miles from North to South, giving an area of

$$A = 3\times10^{6}\ \mathrm{mi}^2 = 6\times10^{6}\ \mathrm{km}^2.$$

Of this, more than 1% and less than 100% is used to grow crops, so we will estimate 10% (in reality, about 20%). This cropland feeds 3×10^{8} Americans, so that each American consumes the produce of

$$a = \frac{0.1(6\times10^{12}\,\mathrm{m}^2)}{3\times10^{8}} = 2\times10^{3}\,\mathrm{m}^2.$$

Thus the land needed to grow our toilet paper is only 10^{-3} of the land needed to grow our food. That is equivalent to skipping one meal per year.

While it takes resources to harvest, transport, sort, pulp, and process trees into toilet paper, it also takes resources to collect, transport, sort, pulp, and process used paper into toilet paper. Let's characterize the energy saved by processing used paper rather than trees in terms of the mass of gasoline needed relative to the mass of the paper produced (using gasoline as a general surrogate for all forms of energy input and environmental impact). The mass of extra gasoline needed will be more than 1% and less than 100% of the mass of paper produced, giving our estimate of 10% of 6 kg or about 0.6 kg of gasoline per person per year.

This is 0.6 L of gasoline, or about enough to drive 5 miles. In other words, the benefit of an entire year's discomfort of using scratchy toilet paper can be gained by forgoing one meal per year and making one fewer trip to the grocery store.

It hardly seems worthwhile.