Fermi Questions

***Larry Weinstein,* Column Editor**
Old Dominion University, Norfolk, VA 23529;
weinstein@odu.edu.

Solutions for Fermi Questions, February 2012

Question 1: Number seven billion

The seven billionth human was born recently. How precisely do we need to measure time to know which specific baby is number seven billion? Is this possible?

Answer: If the population is in a steady state, then there are an equal number of births and deaths each year. Global average life expectancy is more than 50 years and less than 90 years, so we will choose the arithmetically convenient value of 70 years. This means that 1/70 of the population dies each year and thus the death rate is

$$N_{\text{death}} = \frac{7\times10^9}{70\text{ yr}} = (10^8\,\text{yr}^{-1})\frac{1\text{ yr}}{\pi\times10^7\,\text{s}} = 3\text{ Hz}.$$

However, because the population is growing, there are more births than deaths. Let's see if this is significant. The growth rate has been slowing in recent decades and is now only about 1% per year. This means that there are

$$N_{\text{births}} - N_{\text{deaths}} = (10^{-2}\,\text{yr}^{-1})(7\times10^9) = 7\times10^7\,\text{yr}^{-1} = 2\text{ Hz}$$

and thus

$N_{\text{births}} = 5$ Hz.

Thus, in order to determine the identity of the seven billionth human being, we would need to measure the time of birth and the time of death to better than 0.1 s. Measuring time this accurately is easy. The difficulties arise in (a) keeping track of seven billion people with a precision of 1 part in 10^{10}, and (b) defining the biological instants of birth and of death.

The time of birth is not well defined. It can be the time of separation from the mother's body or it can be the time of drawing the first breath. In either case, it will be difficult to determine the time much more precisely than 1 second.

The time of death is even less precise. Death is frequently defined by cessation of breathing. The normal respiration rate is about 10 breaths per minute (more than one and fewer than 100). Since you need to make sure that breathing has definitely stopped, the uncertainty in the time of death will be the time it takes to draw a few breaths, which is on the order of 10 seconds.

However, by far the biggest obstacle to identifying number seven billion is the task of precisely counting seven billion people. The uncertainty in the total population is never specified (alas) but it must be more than 1% and less than 25% so we will estimate 5%. At a global population growth rate of 1%, even a 1% uncertainty implies an uncertainty in the arrival time of the seven-billionth person of one year.

Alas, we will never know the precise identity of number seven-billion.

Question 2: Cost to heat water

How much does it cost to heat the water for a typical American household for one year?

Answer: In order to estimate this, we need to estimate the hot water consumed, the temperature increase of the water, the energy used to heat the water, and the cost of that energy. Here we go.

We can estimate the total water used by adding up showers (10 minutes per day at 3 gallons per minute), dish washing (5 minutes per day at 3 gallons per minute), clothes washing (50 gallons per week), drinking (0.5 gallons per day) and all the other ways we use water. The total is about 50 gallons (200 L) per person per day. About half of this, or 100 L per day, is hot water. There are about three people per household (more than 1 and less than 10) so each household uses about 300 L/day of hot water.

This water is heated from about 60 ºF (winter) or 80 ºF (summer) to about 140 ºF, so that the temperature change is 70 ºF or about 40 ºC (it is also about 40 K, if you prefer). The specific heat of water is just one (in the appropriate units). That is c = 1 cal/g-K or about 4 J/g-K. Thus, the total energy used to heat the water for one household for one year is

$$\begin{aligned} E &= cM\Delta T \\ &= (4\text{ J/gK})(3\times10^2\,\text{L/day})(10^3\,\text{g/L})(40\text{ K})(4\times10^2\,\text{days/yr}) \\ &= (2\times10^{10}\,\text{J})(3\times10^{-7}\,\text{kW-hr/J}) \\ &= 6\times10^3\,\text{kW-hr} \end{aligned}$$

At \$0.1 per kilowatt-hour, this amounts to \$600 per year per household.

This puts a limit on the value of alternative energy systems for heating water. In sunny places, a solar hot water system could heat all of your hot water and save almost

100% of the heating costs. However, in cloudier locations, it might only save you 30%. In terms of capital expenses, if it is worth spending up to $6000 for a solar hot water system in southern California, then it is only worth spending up $2000 for the same system in Boston.