Fermi Questions

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Question 1: Football programs

In the Arthur C. Clarke book, *Tales From the White Hart*, South American soccer fans used sunlight reflected from shiny programs to incinerate a referee. Is this reasonable (physically, not ethically)?

Answer: This question follows naturally from last month's, where we estimated the effects on a parked car of sunlight reflected from a concave building. To estimate this, we'll need to estimate the intensity of the sunlight, the total area of the mirror, the anger level of the fans, and the effective focusing of the mirror. The solar flux outside the atmosphere is 10^3 W/m^2. It is attenuated about 50% by the atmosphere (on a summer sunny day). Each segment of the mirror is the size of a program, which is about 8.5×11 in or 0.2 m×0.25 m = 0.05 m^2. If the Sun is directly overhead so that all the fans can use their programs, then the programs will need to be angled by more than 45°, reducing the effective surface area by about a factor of two. If the Sun is off to one side, then only half the fans can use their programs, also reducing the area by about a factor of two. If it is December in Boston, Chicago, or Seattle, the referee will appreciate the extra warmth.

Let's assume that this is an important game played in a large stadium, with about 6×10^4 fans. Then the total mirror area is

$$A = (6\times10^4 \text{ fans})(5\times10^{-2} \text{ m}^2/\text{fan})/2$$
$$= 10^3 \text{ m}^2$$

and the reflected power is

$$P = (10^3 \text{ m}^2)(5\times10^2 \text{W/m}^2)$$
$$= 5\times10^5 \text{ W}.$$

That is a LOT of power. But it cannot all be focused on the referee.

We can estimate the focusing ability of the crowd as 3% (less than 100% and more than 0.1%), giving 2×10^4 W focused on the referee. That is a lot of power, but can it incinerate the referee?

In round numbers, the referee has a mass of 10^2 kg and a heat capacity equal to that of water, c = 1 cal/g·K = 4×10^3J/kg·K. Thus, the rate of temperature increase of the referee (assuming even heat distribution) will be

$$\dot{T}_{\text{ref}} = \frac{P}{mc} = \frac{2\times10^4\,\text{W}}{(10^2\,\text{kg})(4\times10^3\,\text{J/kg}\cdot\text{K})}\,.$$
$$= 5\times10^{-2}\,\text{K/s}$$

Thus, the referee will not be incinerated, even if he stands still for hours and even if the fans are much better at focusing their mirrors.

However, the power will be delivered to the surface (i.e., to his clothing). The total mass of the clothing is about 1 kg and the heat capacity is about 10 times smaller than that of water. This gives (neglecting reflection and cooling)

$$\dot{T}_{\text{clothing}} = \frac{P}{mc} = \frac{2\times10^4\,\text{W}}{(1\text{ kg})(4\times10^2\,\text{J/kg}\cdot\text{K})}\,.$$
$$= 50 \text{ K/s}$$

The clothing will get very hot very quickly. The referee will undoubtedly start running.

The problem of estimating the rate of temperature increase of a moving target with variable focusing and convective cooling is left as an exercise to the reader.

For more information, see "2.009 Archimedes Death Ray: Testing with MythBusters" at http://web.mit.edu/2.009/www/experiments/deathray/10_Mythbusters.html.

Question 2: Parking area

How much area in the United States is devoted to parking cars?

Answer: To estimate this we need to estimate the parking area needed by one car and the total number of American cars. One parking space has an area of roughly 3 m by 10 m (including the parking space itself and the roadway needed to access it) or 30 m^2. Each car needs two parking spaces, one at home and one at work (or at school or shopping or …). There are 3×10^8Americans, but not all of us own cars, so we will estimate 2×10^8 cars. This means that we need an area

$$A = (30 \text{ m}^2/\text{space})(2 \text{ space/car})(2\times10^8 \text{ cars})$$
$$= 10^{10}\text{m}^2 = 10^4 \text{ km}^2.$$

Wow! That seems like a lot of area. Even if we reduce the amount slightly to include the effects of parking garages, it still seems like a lot.

However, this is not surprising. When we multiply just about anything by 3×10^8 (or even by only 2×10^8), we get a large number. Let's make some reasonable (and unreasonable) comparisons to see the actual scale.

The area of a single city such as New York, LA, or Virginia Beach (this is an example of bathos [anti-climax] comparable to "For God, for country and for Yale") is about 10^3 km^2 so that parking spaces occupy the land area of 10 large cities.

A landfill large enough to store all American trash for a century requires about 10^3 km^2 (see *Guesstimation* for details). Thus we need 10 times more space to store our cars than to store our trash.

The area of the contiguous 48 states is about

$$A_{48} = (5\times10^3 \text{ km})(2\times10^3 \text{ km}) = 10^7 \text{ km}^2,$$

so that parking spaces occupy 0.1% of the contiguous-U.S. land area.

Thus, 10^4 km^2 is definitely a significant amount of area, but it is only a tiny fraction of the area of the United States.