Fermi Questions

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Solutions for Fermi Questions, December 2012

Question 1: Santa's sleigh

In popular mythology, Santa delivers toys to all the Christian children in the world on Christmas Eve. What is the kinetic energy of Santa's sleigh at the beginning of his journey, including all the presents?

Answer: To answer this, we need to estimate the mass and velocity of his sleigh. We will assume that he starts his journey carrying all of the presents, rather than making frequent trips back to the North Pole to replenish his load. There are 7×10^9 people in the world. India, China, and Japan have about one-third of the world's population. Outside of these countries there are two primary religions, Christianity and Islam, sharing the remaining 2/3 of the population. We will estimate that about one-third of the population is Christian and about one-third of those are children. This gives about 10^9 Christian children. If each child receives 1 kg of presents, then Santa's sleigh must carry 10^9 kg = 10^6 tons of presents. We will ignore the details of designing a flying conveyance that can carry the equivalent of 10 aircraft carriers.

Now we need to estimate the velocity. To do this, we will estimate the distance traveled and the time. The 10^9 children live in about 3×10^8 households. These households are separated in space by more than 10 m and less than 10^2 m, giving an estimate of 30 m and a total distance traveled of

$$d = (3\times10^8)(30\text{ m}) = 10^{10}\text{ m}.$$

By following the Earth's rotation, Santa can lengthen "Christmas Eve" to more than 24 hours, giving an elapsed time of about 10^5 s. Then his average velocity will be

$$v = d/t = (10^{10}\text{ min})/(10^5\text{ s}) = 10^5\text{ m/s},$$

and his kinetic energy will be

$$\begin{aligned} KE &= \frac{1}{2}mv^2 = \frac{1}{2}(10^9\,\text{kg})(10^5\,\text{m/s})^2 \\ &= 5\times10^{18}\,\text{J} \\ &= 10^3\ \text{MT}\ , \end{aligned}$$

where 1 megaton (MT) of TNT contains 4×10^{15} J.

This is a best-case scenario, assuming that his sleigh does not have to stop at each house. If it does stop at each house, then his average speed will still be 10^5 m/s, but his maximum speed will be 2×10^5 m/s, and his maximum kinetic energy will be four times greater. He only has a very short time to visit each house:

$$\begin{aligned} t &= (10^5\text{ s})/(3\times10^8\text{ houses}) \\ &= 3\times10^{-4}\text{ s/house.} \end{aligned}$$

Ignoring the time it takes him to descend the chimney, this gives him 10^{-4} s to accelerate from 0 to 4×10^5 m/s. Therefore when he accelerates he will need a motor or engine that can provide a LOT of power:

$$P = E/t = (5\times10^{18}\text{ J})/(10^{-4}\text{ s}) = 5\times10^{22}\text{ W},$$

or about 10^{-4} of the power output of the entire Sun. In addition, he will need brakes that can absorb all that power as he decelerates.

Those reindeer are literally incredible!

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Question 2: Backup cameras

Should we require all new automobiles to have backup cameras so drivers will no longer back over tricycles, pets, or children?

Answer: In order to determine this, we need to estimate the cost of the backup cameras, the numbers of lives they would save, and the value of each life saved. Let's start with the last. While we cannot place a monetary value on human life, we also cannot afford to spend a trillion dollars to save each life. The amount that society can afford to spend to save one life is related to the average lifetime productivity of one person. The average lifetime productivity of an American equals the per capita GDP times the average lifespan. Our per capita GDP today is about (\$15 trillion/year) /(3×10^8 people) = \$$5\times10^4$/year. With a typical lifespan of 75 years, this gives an average lifetime productivity per American of

$$P = (\$5\times10^4/\text{year})(75\text{ years}) = \$4\times10^6.$$

Alternatively, we can afford to spend more than \$$10^5$ and less than \$$10^8$ per life, giving an estimate of 3×10^6.

Now let's estimate the number of lives that would be saved by backup cameras. Automobiles kill about 3×10^4 people per year in the United States. Backup accidents only account for a small fraction of these. Backup accidents kill more than 10 and fewer than 10^3 people per year, giving an estimate of 10^2. (According to kidsandcars.org, an advocacy group, there were 66 back-over and 70 front-over fatalities in 2010.)

Assuming that backup cameras are 100% effective at eliminating backup accidents, the lifetime productivity of the lives saved will be

$$LP = (\$4\times10^6/\text{life})(10^2 \text{ lives/year})$$
$$= \$4\times10^8/\text{year}.$$

This value will be increased by averting injuries as well as deaths and will be decreased because backup cameras will not prevent all backup accidents.

To estimate the costs of all the backup cameras, we need to estimate the cost of a new camera and the number of new cars sold each year. The cost of a reliable backup camera system, including the camera, display screen, software to turn it on "automagically" when the car is shifted into reverse, and installation will cost more than \$100 and less than \$$10^3$ so we will estimate \$300. Including the lifetime system maintenance costs will about double this to \$600.

To estimate the number of new cars sold each year, we need to estimate the total number of cars in the United States and their average lifetime. There are almost as many cars as people so we will estimate 2×10^8 cars. The average automotive lifetime is about 20 years, so there are about 10^7 new cars sold each year.

Putting it all together, the total cost of installing a camera backup system in every new car sold is

$$C = (10^7 \text{ cars/year})(\$6\times10^2/\text{car})$$
$$= \$6\times10^9/\text{year}.$$

This is about 10 times larger than the "value" of the lives saved.

In other words, it would cost \$60 million to save a statistical life by requiring all new cars to have backup cameras. That is 10 times greater than the average lifetime productivity of that same statistical life.

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