# Fermi Questions

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## Solutions for Fermi Questions, April 2012

### Question 1: Google and Pac-Man

On the 30th anniversary of the introduction of the video game Pac-Man, Google created a miniature version of the game and placed it as its logo on its home page. How much total time was wasted in the United States playing this game?

*(Thanks to Daniel Brill of Old Dominion University for suggesting the question.)*

***Answer:*** In order to estimate the time wasted, we need to estimate the number of people who played the game and the average time they each spent playing it. The number of people playing the game in the United States was more than 1% and less than 100% so we will estimate 10%. (Yes, we could try to break the proportion down by estimating the fraction of people who access Google's home page, the fraction of those that notice the logo, the fraction of those that play the game, etc. However, this would probably not improve our estimate in this case.) This means that $3\times10^7$ Americans played the game that day.

Each person who played the game probably spent more than one minute and less than one hour (60 minutes), so we will estimate eight minutes. Thus the total time wasted that day playing Pac-Man was

$$\begin{aligned} t &= (8\text{ min/person})(3\times10^7\text{ people}) = 2\times10^8\text{ min} \\ &= 3\times10^6\text{ hr} \\ &= 3\times10^2\text{ yr.} \end{aligned}$$

This seems like a lot. However, since only 10% of Americans played the game, the average time spent per person was only 0.8 minutes, or less than 0.2% of the work day.



### Question 2: Magnetic levitation

Animal tissue is diamagnetic. How large a magnetic field gradient is needed to levitate frogs or people?

***Answer:*** To answer this, we need to estimate the interaction energy between a magnetic field and animal tissue and then compare this to the interaction energy between the gravitational field and animal tissue. There are two ways to try to estimate this. We can estimate (or remember) the magnetic moment of an atom (the Bohr magneton), the change in that magnetic moment in an applied magnetic field, the energy of interaction, and then the total effect integrated over all the atoms in the body.

On the other hand, we can estimate the change in the magnetic field caused by the magnetic susceptibility of animal tissue and then the change in the magnetic energy caused by the changed magnetic field.

The magnetic susceptibility of diamagnetic materials is very small. It must be less than $10^{-3}$, or we would be able to demonstrate it easily in the classroom and it must be more than $10^{-6}$, or it would be very difficult to observe in the laboratory. Therefore we will estimate that it is about $\chi = -3\times10^{-5}$. This means that the magnetic field will change by a factor of $1 + \chi$.

The energy stored in a magnetic field is

$$E_m = (B^2/2\mu_0)V,$$

where $V$ is the volume and $\mu_0 = 4\pi10^{-7}$ N/A$^2$ is the magnetic permeability of free space. When we place a frog or elephant or person in the magnetic field, the energy will change to

$$\begin{aligned} E'_m &= (B'^2/2\mu_0)V \\ &= VB^2(1+\chi)^2/(2\mu_0), \end{aligned}$$

(where $V$ is now the volume of the object in the field) and the change in the energy will be

$$\Delta E_m = E'_m - E_m = VB^2\chi/\mu_0.$$

The energy stored in a gravitational field is

$$E_g = mgy = \rho Vgy,$$

where $\rho = 10^3$ kg/m$^3$ is the density of water or animals. In order for an object to levitate, the gradients of the gravitational and magnetic energies must be equal:

$$\frac{d(\Delta E_m)}{dy} = \frac{dE_g}{dy}$$

$$2VB\frac{dB}{dy}\frac{\chi}{\mu_0} = \rho Vg$$

or

$$B\frac{dB}{dy} = \frac{\rho g\mu_0}{2\chi}.$$

Now we can just plug in numbers:

$$B\frac{dB}{dy}=\frac{(10^3\,\text{kg/m}^3)(10\ \text{N/kg})(4\pi 10^{-7}\,\text{N/A}^2)}{2(3\times10^{-5})}$$
$$=2\times10^2\,\text{T}^2/\text{m},$$

where the unit conversions have been left as an exercise to the reader.

This means that if $B$ = 10 T, then you can levitate an animal if the magnetic field gradient is $dB/dy$ = 20 T/m = 0.2 T/cm. Needless to say, this will be much easier to achieve for a frog or a mouse, than for an elephant. Note that the actual magnetic susceptibility is about $10^{-5}$, or about three times smaller than we estimated. For more information, see Y. Liu et al., "Magnetic levitation of large water droplets and mice," *Advances in Space Research* **45,** 208 (2010).