Modica Type Estimates and Curvature Results for Overdetermined p-Laplace Problems

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In this talk, we present Modica type estimates for the following overdetermined p-Laplace problem:

$$\begin{cases} \operatorname{div}\left(|\nabla u|^{p-2}\nabla u\right) + f(u) = 0, & \text{in } \Omega, \\ u > 0, & \text{in } \Omega, \\ u = 0, & \text{on } \partial\Omega, \\ \partial_\nu u = -\kappa, & \text{on } \partial\Omega, \end{cases}$$

where $1 < p < +\infty$, $f \in C^1(\mathbb{R})$, $\Omega \subset \mathbb{R}^n$ $(n \geq 2)$ is a C^1 domain (bounded or unbounded), ν denotes the exterior unit normal of $\partial\Omega$, and $\kappa \geq 0$ is a constant. Building on these Modica type estimates, we obtain rigidity results for bounded solutions. In particular, we show that if there exists a nonpositive primitive F of f satisfying $F(0) \geq -(p-1)\kappa^p/p$ (and for $p > 2$ we further assume that if $F(u_0) = 0$, $F(u) = O(|u - u_0|^p)$ as $u \to u_0$), then either the mean curvature of $\partial\Omega$ is strictly negative or Ω is a half-space.