# LEAST ENERGY SOLUTIONS FOR COOPERATIVE AND COMPETITIVE SCHRÖDINGER SYSTEMS WITH NEUMANN BOUNDARY CONDITIONS

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ABSTRACT. In this talk, we consider the nonlinear Schrödinger system

$$\begin{cases} -\Delta u_1 + \lambda_1 u_1 = u_1^3 + \beta u_1 u_2^2 & \text{in } \Omega, \\ -\Delta u_2 + \lambda_2 u_2 = u_2^3 + \beta u_1^2 u_2 & \text{in } \Omega, \\ \frac{\partial u_1}{\partial \nu} = \frac{\partial u_2}{\partial \nu} = 0 & \text{on } \partial\Omega, \end{cases}$$

where $\Omega$ is a bounded domain with $\partial\Omega \in C^2$, and $\lambda_1, \lambda_2, \beta \in \mathbb{R}$.

The interaction parameter $\beta$ distinguishes cooperative and competitive regimes, leading to different variational behaviors. Using variational methods, we prove the existence of least energy solutions in both subcritical and critical dimensions. The analysis combines minimization on Nehari-type manifolds, linking theorems, and minimization over the set of fully nontrivial solutions.