FORWARDS ATTRACTORS FOR NON-AUTONOMOUS LOTKA-VOLTERRA SYSTEMS

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The autonomous Lotka-Volterra model is used to study the evolution of species population from an ecosystem, and the full characterization of the asymptotic behavior of their solutions is well known. In particular, it is possible to give conditions on the coefficients which guarantee the existence of a globally asymptotically stable equilibrium point, in which all the species are present (known as a permanence solution), or, on the contrary, conditions to obtain a stable equilibrium point which possess one or more species extinct. Furthermore, one can construct the full structure of the global attractor associated through the heteroclinics connections between equilibrium points [1].

Passing these results to the non-autonomous situation, i.e. when the parameters are depending on time, is non-trivial, and in general, it is rare to get time dependent invariant compact attracting sets when time goes to $+\infty$. Based on the works of Lazer and Ahmad [2], [3], and Redheffer [4], we consider a non-autonomous Lotka-Volterra system.

First we study the situation where both the matrix of interaction of the species and the vector of intrinsic growth rates are depending on time. We give sufficient conditions to obtain the existence of a globally asymptotically stable solutions, in both the permanence and extinction situations. Once obtained the stable solution, we obtain the exact geometrical structure of the forwards non-autonomous attractor by constructing the heteroclinic connections between the globally stable solution and the semistables ones.

Later, we relax the complexity of the problem having only the vector of intrinsic growth rates depending on time, and we obtain sufficient conditions on the problem parameters for different structures of attractors which leads to understanding different paths of the solutions towards the globally stable one.

This is a joing work with José Antonio Langa, Piotr Kalita and Antonio Suárez.

References

[1] Y. Takeuchi, (1996). *Global asymptotic dynamical properties of Lotka-Volterra systems.* World Scientific Publishing, 1996.

[2] S.Ahmad, A.Lazer. *On the nonautonomous N-competing species Problems.* Applicable Analysis. **57** (1995) 309-323.

[3] S. Ahmad, A. Lazer. *Necessary and sufficient average growth in a Lotka–Volterra system.* Nonlinear Analysis. **34** (1998) 191-228.

[4] R. Redheffer. *Nonautonomous Lotka–Volterra systems. I.* J.Differential Equations. **127** (1996) 519-541.

[5] J. Garcia-Fuentes, J.A. Langa, P. Kalita, and A. Suárez. *Characterization of attractors for non-autonomous Lotka-Volterra cooperative systems*, https://arxiv.org/abs/2301.04955, 2023.