Schiffer's conjecture on flat tori

JOINT WORK WITH T. WETH AND M. M. FALL

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Abstract

A long standing conjecture by Schiffer [3, Problem 80, p. 688] states that if Ω is a bounded and simply connected smooth domain of $\mathbb{R}^{N+1}$, with $N \geq 1$, such that there exist a constant $\mu > 0$ and a solution $u \neq 0$ to following overdetermined Neumann problem

$$(\mathrm{N}_\mu): \begin{cases} \Delta u + \mu u = 0 & \text{in } \Omega, \\ |\nabla u| = 0 & \text{on } \partial\Omega, \\ u = a \neq 0 & \text{on } \partial\Omega, \end{cases}$$

where a is a real constant, then Ω is a ball.

The validity of Schiffer's conjecture in $\mathbb{R}^{N+1}$ has only been derived in some special cases and the problem remains open. In [1, 2], Berenstein and Yang were able to prove that the existence of a solution to (N_μ) with $\mu = \lambda_2$ imply that Ω is a ball. Here $0 < \lambda_1 < \lambda_2 \leq \lambda_3 \leq \cdots$ are the Dirichlet eigenvalues of the Laplacian on Ω counted with multiplicity. In this talk, we address the problem (N_μ) when $\mathbb{R}^{N+1}$ is replaced with $\mathbb{R}^N \times \mathbb{R}/2\pi\mathbb{Z}$. We present a construction via bifurcation theory of nontrivial compact domains $\Omega \subset \mathbb{R}^N \times \mathbb{R}/2\pi\mathbb{Z}$ where (N_μ) is solved for some positive real numbers $\mu > 0$. This provides a counterexample to Schiffer's conjecture on the manifold $\mathbb{R}^N \times \mathbb{R}/2\pi\mathbb{Z}$ endowed with the flat metric.

References

[1] C. Berenstein : An inverse spectral theorem and its relation to the Pompeiu problem, J. Anal. Math. 37 (1980), 128-144.
[2] C. Berenstein, and P.Yang, P.: An inverse Neumann problem, J. Reine Angew. Math. 382 (1987), 1-21.
[3] S.T. Yau : Seminars on Differential Geometry, Ann. of Math. Stud., Princeton University Press, 1992.