

NEC Polygonal Groups and Tessellations *

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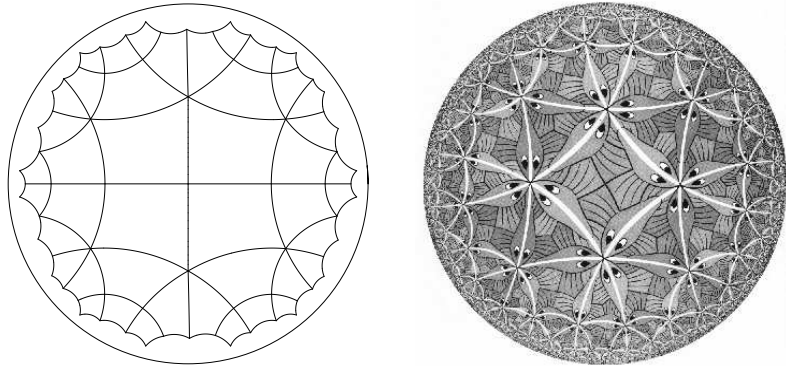
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Abstract

A kaleidoscope is obtained as the quotient of a space by the own discontinuous action of a group of transformations; this can also be obtained from a fundamental domain, which characterizes it. In the present study, the specific case of the Hyperbolic Plane is analyzed with respect to the action of a hyperbolic polygonal group, which is a particular case of an NEC group. Under the action of these groups, the hyperbolic plane is tessellated using tiles with a polygonal shape. The reflections act upon them with respect to their sides as generators of the group. Clear examples of quadrilateral tessellations of the hyperbolic plane with the quadrilaterals of Saccheri and Lambert are given -designed using the *Hyperbol* package created for *Mathematica* software- are found in the basic structure of some of the mosaics of M.C. Escher.

Example 1 Tessellations of the Poincaré disk constructed by M. C. Escher using polygonal groups: *Cirkellimiet I* and *Cirkellimiet III*. In the middle, a tessellation by Saccheri's quadrilaterals.



Example 2 As an example illustrating this situation, we show tessellations of the Poincaré hyperbolic models created with the *Hyperbol* package for *Mathematica* software that it has been developed by the authors ⁽¹⁾: a) On the left, by using a quadrilateral of Lambert. b) On the right, by using the quadrilateral of Saccheri that is obtained from the union of the above quadrilateral of Lambert and its reflect with respect to a side. c) In the middle, the Saccheri quadrilateral thus obtained is the union of three triangles of the type used by Escher for his work *Circle limit IV* known as *Angels and Devils*.



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