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Hyperbolic tessellations by Saccheri and Lambert quadrilaterals

Domingo Gámez, Miguel Pasadas, Rafael Pérez, Ceferino Ruiz*

Dpt. Geometry and Topology, Univ. of Granada, 18071 - Granada, Spain
[ruiz@ugr.es]

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We describe the construction of the Saccheri quadrilateral in the hyperbolic plane using the notion of translation according to a hyperbolic line. Also, we describe the construction of the Lambert quadrilateral using the notion of translation along a line, while establishing the necessary and sufficient condition for the existence and uniqueness of such a quadrilateral. We developed some computer software, in *Mathematica*, oriented to draw the hyperbolic isometries, so much on the Poincaré's half-plane and disk. This software package, named `hyperbol.m`, is available at the Internet address: <http://www.ugr.es/local/ruiz/software.htm>. With this software we have created the graphics which appear in this poster. Finally, the Lambert and Saccheri quadrilaterals that tile the hyperbolic plane, by using triangular Non-Euclidean Crystallographic Groups, are described. And explicit examples of these tessellations are given.

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