Differential Geometry and Global Analysis (session #19)

Special sessions ←

Abstract

Differential Geometry and Global Analysis are two mathematical branches very deeply related between them, which have also deep interrelations with other branches of mathematics, such as Topology, Partial Differential Equations, Calculus of Variations, etc... Many original problems of these areas come from Riemannian Geometry (both extrinsic and intrinsic) and their relations with Topology and Analysis. Along the last decades, there has been a very intensive and fruitful collaboration between Brazil and Spain in the fields of Differential Geometry and Global Analysis, which has allowed to establish a very active net of collaborators including many institutions from both countries.

The main objective of this special session is to encourage the collaboration among the several Spanish and Brazilian research groups on the topic, as well as the exchange of ideas and scientific knowledge among research Spanish and Brazilian groups which share Differential Geometry and Global Analysis as a common area of interest and study.

This session is organized as part of the scientific activites of the Spanish Network of Geometric Analyis (Red Española de Análisis Geométrico, REAG).

Organizers

1. **Alías**, **Luis J.** (Universidad de Murcia, Spain).
2. **Dajczer**, **Marcos** (Instituto de Matemática Pura e Aplicada (IMPA), Rio de Janeiro, Brazil).
3. **García-Río**, **Eduardo** (Universidad de Santiago de Compostela, Spain).
4. **de Lira**, **Jorge H.S.** (Universidade Federal do Ceará, Fortaleza, Brazil).

Ordered list of talks

Block 1

Wednesday 12, 16:00–17:30h (aula 2, ground floor)

1. **Dirichlet problems in warped products** (30 minutes).

   Speaker: **Heinonen**, **Esko** (Universidad de Granada, Spain).

   Warped products are manifolds of the form $M \times_\rho \mathbb{R}$ with Riemannian metric $g_M + \rho^2 dt^2$, where g_M is the metric on M and $\rho > 0$ is a smooth function on M. This notion generalises the Riemannian products $M \times \mathbb{R}$, basic example being the hyperbolic space $\mathbb{H}^{n+1} = \mathbb{H}^n \times_{\cosh r} \mathbb{R}$, where r is the Riemannian distance function on $\mathbb{H}^n$ from a fixed origin.

   Existence of a non-singular Killing vector field makes it possible to define the notion of Killing graphs that are the counterpart of graphs of functions in Riemannian products. Then the existence of Killing graphs with prescribed mean curvature can be studied by solving the corresponding Dirichlet problem on domains of M and by solving the asymptotic Dirichlet problem with continuous boundary data at infinity.

   I will discuss the conditions that guarantee the existence of solutions with prescribed mean curvature in this warped setting. The talk is based on a joint work with J.-B. Casteras, I. Holopainen and J. de Lira.

2. **Nonlinear overdetermined boundary problems in Riemannian manifolds** (30 minutes).

   Speaker: **Domínguez-Vázquez**, **Miguel** (Instituto de Ciencias Matemáticas (ICMAT-CSIC), Madrid, Spain).

   In the context of elliptic PDE theory, an overdetermined boundary value problem is one for which both Dirichlet and Neumann conditions are imposed simultaneously on the boundary of a domain. In a seminal paper in 1971, J. Serrin proved that the only bounded domains of the Euclidean space that admit solutions to certain overdetermined elliptic problems are balls. Since then, multiple extensions of this result have been obtained, some of them showing intriguing connections with the theories of constant mean curvature and isoparametric hypersurfaces.

   One of the still outstanding questions in the area is whether, in a general Riemannian manifold, there are domains that admit solutions to overdetermined problems for a large class of nonlinear elliptic equations. Another open problem is to decide whether the analog of Serrin's result for the rank one symmetric spaces holds or not.

   In this talk I will report on a recent joint work with A. Enciso and D. Peralta-Salas where we provide a positive answer to the first question. I will also explain how our techniques allow us to make progress on the second-mentioned problem, by obtaining certain symmetry results in harmonic spaces and Riemannian manifolds with a large isometry group.

3. **Critical metrics of the volume functional on compact manifolds with boundary** (30

minutes).

Speaker: **Ribeiro Jr.**, **Ernani** (Universidade Federal do Ceará, Fortaleza, Brazil).

In this talk we discuss the space of Riemannian metrics on compact manifolds with boundary that satisfies a critical point equation associated with a boundary value problem. This subject is related to the general question of finding canonical metrics on manifolds with boundary. We present an estimate to the area of the boundary and an isoperimetric inequality for critical metrics of the volume functional on compact manifolds with boundary. In addition, we show that Bach-flat critical metrics of the volume functional on a compact manifold with boundary must be isometric to a geodesic ball in a space form.

Total duration of this block .... **90** minutes.

Block 2

Wednesday 12, 18:00–19:30h (aula 2, ground floor)

1. **Mean curvature flow solitons** (45 minutes).

   Speaker: **Rigoli**, **Marco** (Università degli Studi di Milano, Italy).

   We introduce a general notion of mean curvature flow soliton in general ambient spaces, and we study the particular case of warped product targets. We focus our attention on rigidity results , uniqueness and spectral aspects of their geometry, finally we provide a gradient estimate for translational solitons. This is part of our joint work with L.J. Alías and J.H.S. de Lira, and with G. Colombo and L. Mari.

2. **Graphical translators for mean curvature flow** (45 minutes).

   Speaker: **Martin**, **Francisco** (Universidad de Granada, Spain).

   In this talk we show a full classification of complete translating graphs in $\mathbf{R}^3$. We also construct an $(n-1)$-parameter family of new examples of translating graphs in $\mathbf{R}^{n+1}$.

Total duration of this block .... **90** minutes.

Block 3

Thursday 13, 16:00–17:30h (aula 2, ground floor)

1. **Singularities in the second order geometry of 3-manifolds immersed in** $\mathbb{R}^n$ (30 minutes).

   Speaker: **Romero Fuster**, **M. Carmen** (Universidad de Valencia, Spain).

   The second fundamental form of a 3-manifold M immersed in $\mathbb{R}^n$, determines at each point p of M a surface contained in the normal subspace N_pM, that we call the curvature locus of M at p. This surface has singularities and may present different topological types. We describe all its possible topological types and analyse the connections between the structure of the curvature locus and some of the the extrinsic geometrical properties (local behaviour of principal configurations, local convexity, etc.) at each point of the 3-manifold.

2. **Multiple solutions for an Allen-Cahn type equation and CMC hypersurfaces** (30 minutes).

   Speaker: **Piccione**, **Paolo** (Universidad de São Paulo, Brazil).

   I will discuss some recent results about the multiplicity of solutions for an Allen-Cahn type equation, obtained using classical variational methods. Such solutions are related to constant mean curvature closed hypersurfaces in Riemannian manifolds. Joint work with S. Nardulli (ABC, Brazil), and V. Benci (Pisa, Italy).

3. **Cohomogeneity one actions on Minkowski spaces** (30 minutes).

   Speaker: **Díaz-Ramos**, **José Carlos** (Universidad de Santiago de Compostela, Spain).

   In this talk I will present some results related to the study of cohomogeneity one actions on Minkowski spaces. It is not assumed that the action of the group acting upon is proper. There are examples of orbits that are not closed, and that are not open parts of algebraic hypersurfaces.

Total duration of this block .... **90** minutes.

Block 4

Thursday 13, 18:00–19:30h (aula 2, ground floor)

1. **Geodesic completeness of homogeneous affine surfaces** (45 minutes).

   Speaker: **Gilkey**, **Peter B.** (University of Oregon, USA).

   If γ is a curve in an affine surface $\mathscr{M} = (M,\nabla)$, then the geodesic equation is ${\ddot{\gamma}}^{i} + \Gamma_{jk}^{i}{\dot{\gamma}}^{j}{\dot{\gamma}}^{k} = 0$ for all i. There are 3 families of homogeneous affine surfaces. The Type $\mathscr{A}$ are the left invariant structures on the translation group $\mathbb{R}^2$, the Type $\mathscr{B}$ are the left invariant structures on the $ax+b$ group, and the Type $\mathscr{C}$ arise as the underlying affine structure of a surface of constant sectional curvature. We discuss complete classification results for the geodesically complete Type $\mathscr{A}$ and partial results for the Type $\mathscr{B}$ surfaces. This is work in progess with Daniela D'Ascanio and Pablo Pisani (Dept. Fisica Universidad Nacional de la Plata, Argentina) and X. Valle-Regueiro (Universidade de Santiago, Spain).

2. **The vectorial Ribaucour transformation for submanifolds of constant sectional curvature** (45 minutes).

Speaker: **Tojeiro**, **Ruy** (Universidade Federal de São Carlos, Brazil).

In this talk, I will report on recent joint work with D. Guimarães, in which we have obtained a reduction of the vectorial Ribaucour transformation that preserves the class of submanifolds of constant sectional curvature of space forms. A decomposition theorem for that transformation will be presented, which is a far-reaching generalization of the classical permutability formula for the Ribaucour transformation of surfaces of constant curvature in Euclidean three space. It will be shown how it yields a Bianchi-cube theorem, which allows to produce, from k initial scalar transforms of a given submanifold of constant curvature, a whole k-dimensional cube all of whose remaining $2^k - (k+1)$ vertices are submanifolds with the same constant sectional curvature given by explicit algebraic formulae.

Total duration of this block .... **90** minutes.