A GOMPERTZ-TYPE DIFFUSION PROCESS FOR THE STUDY OF SOCIAL PHENOMENA

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We propose a growth stochastic model associated to the Gompertz curve (bounded and S-shaped growth curve) that improves, in certain aspects, other models in the literature.

Growth is relevant to many applications. For this reason, many mathematical models have been proposed. For instance, Capocelli and Ricciardi [1] proposed a diffusion process associated with the Gompertz curve, including a noise term in the differential equation associated to the deterministic model. On the other hand, Tan [2] defined the stochastic Gompertz birth-death process as that whose mean function is a Gompertz curve. The curves considered in these two works have different expressions; the essential difference between them is its limiting value. For the first one, this is independent of the initial value x_0, but not for the second one.

There are many practical situations in the Social Sciences in which phenomena may be modeled by dynamic variables, with continuous state space, following a Gompertz pattern growing whose limit value depends on x_0. Following the methodology in [1], we present here a new diffusion process whose mean function is a Gompertz curve, a property which is not fulfilled by the above diffusion process. This has the mentioned feature, and it may well be used for forecasting aims.

The resulting process is a lognormal diffusion process with an exogenous factor of the kind $h(t) = \mathrm{e}^{-\beta t}$, for which the known inferential results [3 and references herein] are not valid.

Our work also involves an inferential study of the process as well as an application to real data concerning the salaries along several years in the different Spanish communities. This application allows one to find groups with similar behavior patterns.

[1] Capocelli, R.M., and Ricciardi, L.M., Kybernetik, **15**, 147-154 (1974).

[2] Tan, W.Y. Statistics & Probability Letters, **4**, 25-28 (1986).

[3] Gutiérrez, R., Román, P., Romero, D. and Torres, F., Cybernetics and Systems, **34(8)**, 709-724, (2003).