Approximate and generalized confidence bands for the lognormal diffusion process mean function: a simulation study

Ramón Gutiérrez Jáimez
Universidad de Granada
Departamento de Estadística e Investigación Operativa. Universidad de Granada, Spain.
rgjaimez@ugr.es

Nuria Rico Castro
Universidad de Granada
Departamento de Estadística e Investigación Operativa. Universidad de Granada, Spain.
nrico@ugr.es

Patricia Román Román
Universidad de Granada
Departamento de Estadística e Investigación Operativa. Universidad de Granada, Spain.
proman@ugr.es

Francisco de Asís Torres Ruiz
Universidad de Granada
Departamento de Estadística e Investigación Operativa. Universidad de Granada, Spain.
fdeasis@ugr.es

ABSTRACT

A confidence band for the mean and mode functions of the lognormal difussion process can be calculated following [3], but it implies to get quantiles by solving some integrals that must be tackled using numerical integration. Therefore, they are difficult to use, requiring extensive tables. It is necessary either to have quantile tables, with the restriction of available values which go with it, or to have computational programs. The problem is that these programs are based on numerical algorithms and they are unstable for some ranges of values.

This paper shows different methods to obtain approximate and generalized confidence bands for the lognormal diffusion process mean and mode functions. We have obtained them by generalizing the existing methods for building lognormal distribution mean confidence intervals.

In 1972, Land [6] did a classification in transformation and direct methods. We underline *naive* method and Patterson's transformation inside the transformation methods. Inside the direct methods, we emphasize Cox's method. In 1988 Angus [1] proposed a method known like Angus conservative method. Subsequently, in 1994, Angus [2] proposed a parametric bootstrap method. Moreover, we propose a new method in [7] to obtain an approximate confidence interval.

In 2003, Krishnamoorthy and Mathew [5] obtained a generalized confidence interval for the lognormal distribution mean, based on the concepts of pivotal generalized quantity and generalized confidence interval due by Weerahandy [8].

We extend all these methods in the same sense of [4] to obtain approximate and generalized confidence bands for the aforementioned functions.

Finally, we have realized a simulation study to compare the obtained confidence bands from simulated sample paths of the homogeneous lognormal diffusion process. This study is similar to the one realized by Zhou and Gao [9] for the mean of the lognormal distribution based on simulated observations. The comparisons have been done in terms of coverage probabilities, error probabilities and average widths.

The simulation study finds that *naive* method is inappropriate and that bootstrap, Patterson and proposed methods have a good behavior, excepting the last one for large values of σ^2. Angus conservative method gives coverage probabilities higher than the fixed confidence level, Cox's method gives better results when sample size increases, and it becomes similar to Patterson's one. Generalized band is a Monte Carlo procedure for calculating proposed band, and its behavior is similar to this one when a sufficient amount of random values are generated for its calculation.

To sum up, we will choose bootstrap method for small sample sizes and Patterson's or Cox's ones for big or small sample sizes. Hence, these methods give lower average widths for all the considered cases.

Keywords: Lognormal diffusion process, Confidence bands, Mean and mode functions.

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