Asymptotics for Magic Squares of Primes

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Abstract. Using the techniques and results recently introduced by Ben Green and Terence Tao (see [1, 2, 3, 4]), we obtain an asymptotic formula for $M_n(x)$, the number of $n \times n$ magic squares with their entries being prime numbers less than or equal to x.

$$M_n(x) \sim \mathfrak{S}_n \frac{x^{n^2-2n}}{\log^{n^2} x},$$

where $\mathfrak{S}_n$ is a constant that involves the volume of the polytope associated with the system of equations that define the magic square and an infinite product of local factors $\prod_p \beta_{p,n}$.

For example, in the case $n = 3$ we obtain

$$M_3(x) \sim \mathfrak{S}_3 \frac{x^3}{\log^9 x}$$

where

$$\mathfrak{S}_3 = \frac{243}{8} \prod_{p \geq 5} \left(1 - \frac{6}{p-1} + \frac{13}{(p-1)^2}\right) \left(1 + \frac{1}{p-1}\right)^6 \approx 25,818.$$

This is the first application of the work of Ben Green, Terence Tao and Tamar Ziegler on linear equations in primes with complexity greater than 2, obviously apart from the case of arithmetic progressions. We must point out that even the existence of infinitely many magic squares with prime entries was unknown till very recently.

References

[1] Green, B.; Tao, T. An inverse theorem for the Gowers U^3-norm. *Proc. Edinburgh Math. Soc.* **51** (2008), no. 1, 73–153.

[2] Green, B.; Tao, T. Linear equations in primes. *Annals of Math.* **171** (2010), no. 3, 1753–1850.

[3] Green, B.; Tao, T. The Möbius function is strongly orthogonal to nilsequences. *Annals of Math.* **175** (2012), no. 2, 541–566.

[4] Green, B.; Tao, T.; Ziegler, T. An inverse theorem for the Gowers $U^{s+1}[N]$-norm. *Annals of Math.* **176** (2012), no. 2, 1231–1372.