Asymptotic regularity of the composition of firmly nonexpansive mappings

Victoria Martín Márquez

Victoria Martín Márquez (victoriam@us.es)
Universidad de Sevilla

Abstract. Because of Minty's classical correspondence between firmly nonexpansive mappings and maximally monotone operators, the notion of firm nonexpansivity has proven to be of basic importance in fixed point theory, monotone operator theory, and convex optimization. We show that if finitely many firmly nonexpansive mappings defined on a real Hilbert space are given and each of these mappings is asymptotically regular, which is equivalent to saying that they have or "almost have" fixed points, then so ir their composition. This significantly generalizes the result by Bauschke [1] for the case of projectors (nearest point mappings). The proof resides in a Hilbert product space and it relies upon the Brezis-Haraux range approximation Theorem [3], well-known in Operator Theory.

Joint work with H.H. Bauschke, S.M. Moffat and X. Wang [2].

References

[1] Bauschke, H.H. The composition of finitely many projections onto closed convex sets in Hilbert space is asymptotically regular, *Proceedings of the American Mathematical Society* 131 (2003), 141–146.

[2] Bauschke, H.H.; Martín-Márquez, V.; Moffat, S.M.; Wang, X. Compositions and convex combinations of asymptotically regular firmly nonexpansive mappings are also asymptotically regular, *Fixed Point Theory and Applications*, 2012:53, doi:10.1186/1687-1812-2012-53.

[3] Brézis, H.; Haraux, A. Image d'une somme d'opérateurs monotones et applications, *Israel Journal of Mathematics*, **23** (1976), 165–186.