Resolvent series and C_0-semigroups defined by Laguerre expansions

Luciano Abadias and Pedro J. Miana

Luciano Abadias (labadias@unizar.es)
Universidad de Zaragoza & IUMA

Pedro J. Miana (pjmiana@unizar.es)
Universidad de Zaragoza & IUMA

Abstract. Let $\alpha > -1$, the Laguerre generalized polynomials are given by

$$L_n^{(\alpha)}(t) = e^t \frac{t^{-\alpha}}{n!} \frac{d^n}{dt^n}(e^{-t} t^{n+\alpha}), \qquad t \in \mathbb{R}^+.$$

Some functions defined in $\mathbb{R}^+$ can be expanded in series of Laguerre polynomials,

$$f(t) = \sum_{n=0}^{\infty} c_n \, L_n^{(\alpha)}(t),$$

see [2]. We extend this result for vector-valued functions in a Banach space, obtaining expansions for certain families of operators as C_0-semigroups and resolvent families. In particular, we show that

$$e^{-tA}(x) = \sum_{n=0}^{\infty} \frac{A^n}{(1+A)^{n+\alpha+1}}(x) L_n^{(\alpha)}(t)$$

where $x \in D(A)$ and $(e^{-tA})_{t\geq 0}$ is a C_0-uniformly bounded semigroup generated by $(A, D(A))$. This approach completes similar one shown in [1] .

References

[1] Grimm, V; Gugat, M. Approximation of semigroups and related operator functions by resolvent series. *SIAM Journal on Numerical Analysis* **48,** no. 5. (2010), 1826–1845.

[2] Lebedev, N. N. Special functions and their applications. *Selected Russian Publications in the Mathematical Sciences. Prentice-Hall* (1965).