A unified approach to dispersion and unique continuation for Schrödinger equations

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Abstract. In this talk we survey some recent results on the dynamics of linear, variable-coefficient Schrödinger equations that can be described in terms of high-frequency solutions (for instance, the Schrdinger group on a compact Riemannian manifold).

A typical question that can be answered in those terms is that of characterizing weak limits of quadratic quantities such as the position densities $\left|e^{it\Delta}u_n(x)\right|^2$ when (u_n) is a L^2-bounded sequence of initial data. This has implications in the study of dispersive effects (quantified in terms of Stricharz estimates) or in the study of unique continuation (as described by observability-type estimates). It turns out that the answers to these questions depend of global dynamical properties of the underlying Hamiltonian classical dynamics (for instance, in the case of a manifold they depend on the global dynamics of the geodesic flow).

We shall focus on simple examples, such as the sphere and the torus, were already our approach gives new results on dispersive estimates and unique continuation properties. Our analysis relies on tools of phase-space harmonic analysis, such as semiclassical measures, that are natural when dealing with high-frequency phenomena. Part of the results discussed here were obtained in collaboration with Nalini Anantharaman.