A Gaussian quadrature rule for oscillatory integrals on a bounded interval

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Abstract. We investigate a Gaussian quadrature rule and the corresponding orthogonal polynomials for the oscillatory weight function $e^{i\omega x}$ on the interval $[-1,1]$, where $\omega \geq 0$. Since the weight function is not real, not all orthogonal polynomials $P_n(x)$ exist, and their zeros are not on the interval $[-1,1]$, but distributed in the complex plane. We show that such a rule attains high asymptotic order for large ω, in the sense that the quadrature error quickly decreases as a function of the frequency. As $\omega \to 0$, one recovers the classical Legendre case, as expected.

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