INdAM Meeting: International meeting on numerical semigroups

Cortona 2014

Organizers:
M. D'Anna, Università degli Studi di Catania
P. A. García-Sánchez, Universidad de Granada
V. Micale, Università degli Studi di Catania

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José Carlos Rosales, Universidad de Granada

September, 8-13
Il Palazzone, Cortona, Italy

Schedule

	Monday	Tuesday	Wednesday	Thursday	Friday
09:30	Opening	Watanabe	Ruano	Geroldinger	Branco
10:00	Bryant	Numata	Matthews	Martino	García-García
10:30	Stokes	Matsuoka	Farrán	Vigneron Tenorio	Viola
Coffee Break					
11:30	Delgado	Zarzuela	Tamone	Fel	Moreno Frías
12:00	Aguiló-Gost	Di Marca	Ramírez Alfonsín	García Barroso	Eliahou
12:30	Llena	Strazzanti	Bras Amoros	Baginski	Sammartano

Lunch

16:00		Fröberg		Moyano	
16:30		Leamer		Moree	
Coffee Break					
17:30		Márquez		Ciolan	

List of abstracts

Francisco Aguiló-Gost *Universitat Politècnica de Catalunya*

Some comments on denumerant of numerical 3-semigroups

Joint work with Pedro A. García-Sánchez and David Llena

We give some comments and remarks on the denumerant of numerical 3-semigroups $S = \langle a, b, c\rangle$, $a < b < c$, $\gcd(a, b, c) = 1$. We use these remarks for deriving an efficient algorithm with time cost ranging from $O(1)$ to $O(c)$. Closed expressions of denumerant can be obtained under certain conditions.

Paul Baginski *Smith College, Northampton*

Elasticity in Numerical Semigroup Rings and Power Series Rings

Given a numerical semigroup S and a field K, we can construct the numerical semigroup ring $K[S] \subseteq K[x]$, consisting of polynomials with coefficients in K where the only powers of x permitted are those integers that appear in S. Similarly, one can construct the numerical semigroup power series ring $K[[S]]$. Both of these integral domains have nonunique factorization, yet their behaviors are drastically different. We will examine the factorization properties of these rings under the light of the elasticity. Given a nonzero nonunit x, the elasticity $\rho(x)$ is the length of a longest factorization of x, divided by the shortest factorization length of x. The elasticity of the ring R is $\rho(R) = \sup_{x\in R} \rho(x)$. We will discuss the value of $\rho(R)$ and the distribution of elasticities of elements in $K[S]$ and $K[[S]]$, especially in how these questions relate to the analogous answers for S itself.

Manuel B. Branco *Universidade de Évora*

The Frobenius problem for Mersenne numerical semigroups

A numerical semigroup is a subset S of $\mathbb{N}$ that is closed under addition, contain 0 and has finite complement in $\mathbb{N}$.

Given a nonempty subset A of $\mathbb{N}$ we will denote by $\langle A\rangle$ the submonoid of $(\mathbb{N}, +)$ generated by A, that is,

$$\langle A\rangle = \{\lambda_1 a_1 + \cdots + \lambda_n a_n \mid n \in \mathbb{N}\backslash\{0\}, a_i \in A, \lambda_i \in \mathbb{N} \text{ for all } i \in \{1, \ldots, n\}\}.$$

It is well known (see for instance [4]) that $\langle A\rangle$ is a numerical semigroup if and only if $\gcd(A) = 1$.

A positive integer x is a Mersenne number if $x = 2^n - 1$ for some $n \in \mathbb{N}\backslash\{0\}$. We say that a numerical semigroup S is a Mersenne numerical semigroup if there exist $n \in \mathbb{N}\backslash\{0\}$ such that $S = \left\langle\left\{2^{n+i} - 1 \mid i \in \mathbb{N}\right\}\right\rangle$.

The main purpose in this talk is to study this class of numerical semigroups and will be denoted by $S(n) = \left\langle\left\{2^{n+i} - 1 \mid i \in \mathbb{N}\right\}\right\rangle$.

We will give formulas for the embedding dimension, the Frobenius number, the type and the genus for a numerical semigroups generated by the Mersenne numbers greater than or equal to a given Mersenne number.

Joint work with J.C.Rosales and D. Torrão.

References

[1] V. Barucci, D. E. Dobbs and M. Fontana, " Maximality Properties in Numerical Semigroups and Applications to One-Dimensional Analytically Irreducible Local Domains", Memoirs of the Amer. Math. Soc. 598 (1997).

[2] J. L. Ramírez Alfonsín, "The Diophantine Forbenius Problem", Oxford University Press, London (2005).

[3] J. C. Rosales and M.B. Branco, Numerical semigroups that can be expressed as an intersection of symmetric numerical semigroups, J. Pure and Applied Algebra, 171 (2002), 303-314.

[4] J. C. Rosales, P. A. García-Sánchez, " Numerical semigroups ", Developments in Mathematics, vol.20, Springer, New York, (2009).

[5] E. S. Selmer, On linear diophantine problem of Frobenius, J. Reine Angew. Math., 293/294 (1977), 1-17.

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Maria Bras-Amorós *Universitat Rovira i Virgili*

THE ORDINARIZATION TRANSFORM OF A NUMERICAL SEMIGROUP AND SEMIGROUPS WITH A LARGE NUMBER OF INTERVALS

A numerical semigroup is said to be ordinary if it has all its gaps in a row. Indeed, it contains zero and all integers from a given positive one. One can define a simple operation on a non-ordinary semigroup, which we call here the ordinarization transform, by removing its smallest non-zero non-gap (the multiplicity) and adding its largest gap (the Frobenius number). This gives another numerical semigroup and by repeating this transform several times we end up with an ordinary semigroup. The genus, that is, the number of gaps, is kept constant in all the transforms.
This procedure allows the construction of a tree for each given genus containing all semigroups of that genus and rooted in the unique ordinary semigroup of that genus. We study here the regularity of these trees and the number of semigroups at each depth. For some depths it is proved that the number of semigroups increases with the genus and it is conjectured that this happens at all given depths. This may give some light to a former conjecture saying that the number of semigroups of a given genus increases with the genus.
We finally give an identification between semigroups at a given depth in the ordinarization tree and semigroups with a given (large) number of gap intervals and we give an explicit characterization of those semigroups.

Lance Bryant *Shippensburg University*

POSITION VECTORS OF NUMERICAL SEMIGROUPS

We provide a new way to represent numerical semigroups by showing that the position of every Apéry set of a numerical semigroup S in the enumeration of the elements of S is unique, and that S can be re-constructed from this "position vector." We extend the discussion to more general objects called numerical sets, and show that there is a one-to-one correspondence between m-tuples of positive integers and the position vectors of numerical sets closed under addition by $m + 1$. We consider the following problems:

(1) Determining which position vectors correspond to numerical semigroups.
(2) Counting numerical sets with fixed invariants.
(3) Determining which position vectors correspond to numerical semigroups with embedding dimension 2.

This is joint work with James Hamblin.

Alexandru Ciolan *Jacobs University, Bremen*

CYCLOTOMIC NUMERICAL SEMIGROUPS

The purpose of this talk is to introduce the concept of *cyclotomic numerical semigroups*. We say that a numerical semigroup S is cyclotomic if the roots of its semigroup polynomial $P_S(x) = (1-x)\sum_{s\in S} x^s$ lie in the unit disk. We prove that cyclotomic numerical semigroups are symmetric and give examples when the converse is true or false. We prove that complete intersection numerical semigroups are cyclotomic and conjecture the converse on providing computer evidence with `GAP`. Further, we say that a numerical semigroup S is cyclotomic of *depth* d and *height* h if d and h are chosen minimally, that is, $P_S(x)|(x^d-1)^h$ but $P_S(x)$ does not divide $(x^n-1)^{h-1}$ for any n and does not divide $(x^{d_1}-1)^h$ for any $d_1 < d$. We classify the cyclotomic numerical semigroups of certain prescribed depths and heights.
Joint work with P. A. García-Sánchez and P. Moree.

Manuel Delgado *Universidade do Porto*

Experiments with Numerical Semigroups

The main ingredient of my talk is the GAP package numericalsgps [1]: a package to compute with numerical semigroups.

It is open-source software. The first version of the package was released in 2006 and has been the result of joint work with Pedro A. García-Sánchez and José Morais. Since then it has been extended and maintained jointly with Pedro. In what concerns extension, several functions have been added; some of them implemented by Alessio Sammartano. The maintenance comprises fixing bugs (the ones that may lead to wrong results force immediately a new release) and replacing the implemented algorithms by more efficient ones whenever they come to our knowledge.

It is my intention to do some experiments, ranging from simple ones to other more sophisticated or involving some programming. I plan to show some of the results of the computations by using pictures made with the help of the GAP package intpic [2].

References

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[2] M. Delgado, P. A. García-Sánchez and J. Morais, "NumericalSgps", *A GAP package for numerical semigroups*. Available via `http://www.gap-system.org/`.

Michela Di Marca *Università di Catania*

On the Hilbert function of one-dimensional semigroup rings

In this talk I will explain some sufficient and necessary conditions for a generic one-dimensional semigroup ring in order to have decreasing or non-decreasing Hilbert function.

To this aim I will intoduce certain invariants of the semigroup, with particular regard to its Apéry-set.

Shalom Eliahou *Université du Littoral Côte d'Opale*

Some remarks on Wilf's conjecture

Let S be a numerical semigroup with conductor c. Let L be the 'left part' of S, that is, the intersection of S with the integer interval $I = [0, c-1]$. A few decades ago, Wilf provided a conjectural lower bound for the density of L inside I, i.e. for $\mathrm{card}(L)/c$. Among other particular cases, Wilf's conjecture is known to hold whenever $\mathrm{card}(L)$ is at most 4. We shall revisit the known proof of this result and extend it to slightly higher values of $\mathrm{card}(L)$.

José Ignacio Farrán *Universidad de Valladolid*

Numerical semigroups and codes

The parameters of one-point algebraic geometry codes are related to a certain Weierstrass semigroup, by means of the so-called Feng-Rao distances. For the classical (unidimensional) distance, strong results are known, but the problem becomes hard for the mutidimensional case. The asymptotical behaviour of such generalized Feng-Rao distances is related by the so-called Feng-Rao number. We compute these numbers for some special cases of numerical semigroups, like those generated by intervals or with embedding dimension two.

This is a joint work with M. Delgado, P. A. Garcia-Sanchez, and D. Llena.

Leonid Fel *Technion - Israel Institute of Technology*

On numerical experiments with symmetric semigroups generated by three elements and their generalization

We give a simple explanation of numerical experiments of V. Arnold with two sequences of symmetric numerical semigroups, $\langle 4, 6+4k, 87-4k\rangle$ and $\langle 9, 3+9k, 85-9k\rangle$ generated by three elements. We present a generalization of these sequences by numerical semigroups $\langle r_1^2, r_1r_2 + r_1^2k, r_3 - r_1^2k\rangle$, $k \in Z$, $r_1, r_2, r_3 \in \mathbb{Z}_+$, $r_1 \geq 2$ and $\gcd(r_1, r_2) = \gcd(r_1, r_3) = 1$, and calculate their Frobenius number $\phi(r_1, r_2, r_3)$ for the wide range of k providing semigroups be symmetric.

We show that this type of semigroups admit also nonsymmetric representatives. We describe the reduction of the minimal generating sets of these semigroups up to $\{r_1^2, r_3 - r_1^2k\}$ for sporadic values of k and find these values by solving the quadratic Diophantine equation.

Ralf Fröberg *Stockholms Universitet*

On free resolutions of some semigroup rings

For some numerical semigroup rings of small embedding dimension, namely those of embedding dimension 3, and symmetric or pseudosymmetric of embedding dimension 4, presentations has been determined in the literature. We extend these results by giving the whole graded minimal free resolutions explicitly. Then we use these resolutions to determine some invariants of the semigroups and certain interesting relations among them. Finally, we determine semigroups of small embedding dimensions which have strongly indispensable resolutions.

Evelia R. García Barroso *Universidad de la Laguna*

On the Abhyankar-Moh inequality

Let C be a complex affine alebraic curve of degree $n > 1$ having only one branch at infinity γ and let $r_0, r_1, \ldots, r_h$ be the n-sequence of the semigroup G of the branch γ defined as follows: $r_0 = n$, $r_k = \min\{r \in G \mid r \notin \mathbb{N}r_0 + \cdots + \mathbb{N}r_{k-1}\}$ for $k \geq 1$ and $G = \mathbb{N}r_0 + \cdots + \mathbb{N}r_h$.

Then the Abhyankar-Moh inequality (see [1, 2]) can be stated in the form

(AM_n) $\qquad \gcd(r_0, \ldots, r_{h-1})r_h < n^2.$

The aim of this talk is to present (see [3]) some results on the semigroups $G \subset \mathbb{N}$ on plane branches γ with property (AM_n). In particular we describe such semigroups with the maximum conductor.

References

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Juan Ignacio García García *Universidad de Cádiz*

On Delta set of numerical monoids

This is a Joing work with M. A. Moreno Frías and A. Vigneron-Tenorio.

Given the minimal generating set, $\{a_1, \ldots, a_p\}$, of a numerical monoid S and any element s belonging to S, the Delta set of s is defined by $\Delta(s) = \{l_i - l_{i-1} \mid i = 2, \ldots, k\}$ where $\{l_1 < \cdots < l_k\}$ is the set $\{\sum_{i=1}^{p} x_i \mid s = \sum_{i=1}^{p} x_i a_i$ and $x_i \in \mathbb{N}$ for all i$\}$. In addition, the Delta set of S is the union of $\Delta(s)$ of all $s \in S$. As proved in [1], it is only necessary to compute the Delta set of a finite number of elements to compute $\Delta(S)$; this number of elements is determined by a bound. In this paper, by using geometrical tools, we obtain a new bound for the computation of $\Delta(S)$. This bound is compared with the obtained in [1] and an algorithm to compute $\Delta(S)$ from the factorizations of only a_1 elements is given.

References

[1] S.T. Chapman, R. Hoyer, and N. Kaplan, *Delta sets of numerical monoids are eventually periodic,* Aequationes Mathematicae **77**(2009) 273-279.

Alfred Geroldinger *Karl-Franzens-Universität Graz*

Arithmetic of seminormal weakly Krull monoids and domains

We study the arithmetic of seminormal v-noetherian weakly Krull monoids and domains with nontrivial conductor which have finite class group and prime divisors in all classes. These monoids include seminormal orders in holomorphy rings in global fields. If H is such a domain, then the monoid $\mathcal{I}_v^*(H)$ of v-invertible v-ideals is isomorphic to a coproduct of localizations $\coprod_{\mathfrak{p}\in\mathfrak{X}(H)}(H_\mathfrak{p})_{\mathrm{red}}$. The localizations $H_\mathfrak{p}$ are finitely primary monoids which are a sort of higher-dimensional numerical monoids (indeed, numerical monoids are finitely primary of rank one).

The crucial property of seminormality allows us to give precise arithmetical results analogous to the well-known results for Krull monoids having finite class group and prime divisors in each class. This allows us to show, for example, that unions of sets of lengths are intervals and to provide a characterization of half-factoriality.

This is joint work with Florian Kainrath and Andreas Reinhart [1].

References

[1] A. Geroldinger, F. Kainrath, and A. Reinhart, *Arithmetic of seminormal weakly Krull monoids and domains*, submitted.

Micah J. Leamer *Univeristy of Puerto Rico*

Identifying torsion in the tensor product of modules by analyzing factorization properties of related semigroups

Let Γ be a numerical semigroup and let $s \in \mathbb{N} \setminus \Gamma$. We consider the semigroup of finite arithmetic sequences with stepsize s and setwise addition, denoted $S_\Gamma^s := \{\{z, z+s, \ldots, z+ns\} \subset \Gamma \mid\ n \geq 1\}$. We would like to be able to show that S_Γ^s always contains an element $\{z, z+s, z+2s\}$ which is an atom, i.e. it does not factor as $\{x, x+s\}+\{y, y+s\}$. It turns out that the existence of such an element for any s is equivalent to showing that two generated monomial ideals over the numerical semigroup ring $K(\Gamma)$ satisfy the Huneke-Wiegand Conjecture. The Huneke-Wiegand Conjecture is an open problem in ring theory that asserts that torsion always exists in the tensor product of certain types of modules. We show how progress is being made on the problem for our special case. Then we explore possibilities for translating the conjecture into factorization properties of semigroups when the ideal in question has more than two generators.

David Llena *Universidad de Almería*

Hyper L-shapes for an embedding dimension four numerical semigroup. How many there are?

This is a joint work with F. Aguiló-Gost and P. A. García-Sánchez

Associated to the Minimum Distance Diagrams and, as a particular case of them, one can associate L-shaped tiles to embedding dimension three numerical semigroup. They tessellate the plane, and have proven to be useful in the study and computation of some properties of numerical semigroups with embedding dimension three (see the references below). Associated to an embedding dimension three numerical semigroup we can have at most two different L-shapes.

When we try to generalize this idea to higher dimensions, we find several problems. One of these is that, there is in general more than two L-shapes associated to the numerical semigroup, because working in the three dimensional space, several different positions can be used for some unitary cubes. In this work, we try to compute how many different L-shapes can be associated to an embedding dimension four numerical semigroup.

References

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[2] F. Aguiló-Gost, A. Miralles and M. Zaragozá, Some contributions to the Frobenius' Problem, *Electronic Notes in Discrete Mathematics.* 28, 61-68. 2007.

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Guadalupe Márquez Campos *Universidad de Sevilla*

On the computation of the Apéry set of numerical monoids and affine semigroups

A simple way of computing the Apéry set of a numerical semigroup (or monoid) with respect to a generator, using Groebner bases, is presented, together with a generalization for affine semigroups. This computation allows us to calculate the type set and, henceforth, to check the Gorenstein condition which characterizes the symmetric numerical subgroups.

This is a joint work with G. Márquez-Campos and I. Ojeda.

Ivan Martino *Stockholms Universitet*

Syzygies of the Veronese modules

The Veronese ring $S^{(d)}$ of a polynomial ring $S = K[x_1, \ldots, x_n]$ is the semigroup ring generated by all the monomials of S of degree d.The Betti numbers of $S^{(d)}$ can be seen as the syzygies of the related affine semigroup in $\mathbb{N}^n$ and they can be computed via the minimal free resolution of $S^{(d)}$.

In the beginning of this talk, I am going to give a summary of some results on minimal free resolutions of Veronese rings. Then I will concentrate on the Veronese modules. I present a formula for their Betti numbers in terms of some topological invariants of certain simplicial complexes. As applications, I will characterize the Cohen-Macaulayness of $S_{n,d,k}$ and the linearity of the resolution.

Naoyuki Matsuoka *Meiji University*

Almost Gorenstein rings

The notion of an almost Gorenstein ring is introduced by Brucci and Froeberg [1] in 1997 in the case where the base local ring is analytically unramified. In this talk, I will give a new definition of the almost Gorenstein property for any one-dimensional Cohen-Macaulay local ring as in [2]. Moreover, we will see our definition includes Barucci-Froeberg's one and solve the problem when the algebra $\mathfrak{m} : \mathfrak{m}$ is a Gorenstein ring, where $\mathfrak{m}$ denotes the maximal ideal of the base local ring.

References

[1] V. Barucci and R. Fröberg, One-dimensional almost Gorenstein rings, J. Algebra, 188 (1997), 418–442.

[2] Shiro Goto, Naoyuki Matsuoka, Tran Thi Phuong, Almost Gorenstein rings, Journal of Algebra, 379(2013), 355–381.

Gretchen Matthews *Clemson University*

Applications of algebraic geometry codes beyond coding

Weierstass semigroups are an important tool in the study algebraic geometry codes. In this talk, we discuss applications of algebraic geometry codes beyond that of traditional coding theory and the role played by associated numerical semigroups.

Pieter Moree *Max-Planck-Institut für Mathematik*

Cyclotomic coefficients yesterday and tomorrow

Alexandru Ciolan in his talk will introduce the concept of a cyclotomic numerical semigroup. For a good understanding of these some background knowledge on cyclotomic polynomials is crucial. I will provide this in my talk, with a special focus on properties of coefficients of cyclotomic polynomials.

M$^{\mathrm{a}}$ Ángeles Moreno Frías *Universidad de Cádiz*

ω-PRIMALITY AND ASYMPTOTIC ω-PRIMALITY ON NUMERICAL SEMIGROUPS. COMPUTATION AND PROPERTIES

Let S be a monoid, the ω-invariant, introduced in [5], is a well-established invariant in the theory of non-unique factorizations. This invariant essentially measures how far an element of an integral domain or a monoid is from being prime (see [1]). In [4] it is proven that the tame degree and ω-primality coincide for half-factorial affine semigroups and in [2] and [4] the ω-primality is computed for some kinds of affine semigroups (when the semigroup is the intersection of a group and $\mathbb{Z}^p$). Associated with the ω-primality there is its asymptotic version, the asymptotic ω-primality or $\overline{\omega}$-primality, which is object of study in several works.

In this talk we give an algorithm to compute from a presentation of a finitely generated atomic monoid the ω-primality of any of its elements. Also, we give an explicit formulation of the asymptotic ω-primality for finitely generated quasi-Archimedan cancellative monoids(note that every numerical semigroup belongs to this class of monoids), and a method to compute their asymptotic ω-primalities. All the theoretical results of this work are complemented with the software `OmegaPrimality` developed in `Mathematica` (see [3]). This software provides functions to compute the ω-primality of a monoid and its elements from one of its presentations or from a system of generators. The contents of this talk will appear in [6].

References

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[2] V. Blanco, P.A. García-Sánchez and A. Geroldinger, *Semigroup-theoretical characterizations of arithmetical invariants with applications to numerical monoids and Krull monoids*, Illinois J. Math. **55** (2011), no. 4, 1385–1414.

[3] J.I. García-García, A. Vigneron-Tenorio. *OmegaPrimality, a package for computing the omega primality of finitely generated atomic monoids.* Handle: `http://hdl.handle.net/10498/15961`(2014).

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[6] J.I. García-García, M.A. Moreno-Frías and A. Vigneron-Tenorio, *Computation of the ω-primality and asymptotic ω-primality with applications to numerical semigroups.* Israel Journal of Mathematics, to appear.

Julio J. Moyano-Fernández *University Jaume I of Castellón*

COUNTING MODULES OVER NUMERICAL SEMIGROUPS WITH TWO GENERATORS.

Let $\Gamma = \langle \alpha, \beta \rangle$ be a numerical semigroup. In this article we consider several relations between the so-called Γ-semimodules and lattice paths from $(0, \alpha)$ to $(\beta, 0)$: we investigate isomorphism classes of Γ-semimodules as well as certain subsets of the set of gaps of Γ, and finally syzygies of Γ-semimodules. In particular we compute the number of Γ-semimodules which are isomorphic with their k-th syzygy for some k.

This is joint work with Jan Uliczka.

Takahiro Numata *Nihon University*

A VARIATION OF GLUING OF NUMERICAL SEMIGROUPS

We study the relation between two numerical semigroups $S = \langle a_1, ..., a_n \rangle$ and $T = \langle da_1, ..., da_{n-1}, a_n \rangle$, where $d > 1$ and $\gcd(d, a_n) = 1$. When $a_n \in \langle a_1, ..., a_{n-1} \rangle$ and $a_n \neq a_i$ for any $1 \leq i \leq n-1$, this was studied in [Wa] and T is called a gluing of $\langle a_1, ..., a_{n-1} \rangle$ and $\mathbb{N}$ in [RG]. We consider the case where $a_n \notin \langle a_1, ..., a_{n-1} \rangle$. Then we prove that T is never almost symmetric unless it is symmetric. We also prove that the Betti numbers of $k[T]$ and $k[S]$ over respective polynomial rings are the same, where $k[T]$ and $k[S]$ are their semigroup rings.

References

[RG] J. C. Rosales and P. A. García-Sánchez, *Numerical semigroups*, Springer Developments in Mathematics, Volume **20** (2009).

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Jorge Ramírez Alfonsín *Université Montpellier 2*

MÖBIUS FUNCTION OF SEMIGROUP POSETS THROUGH HILBERT SERIES

In this talk we will investigate the Möbius function $\mu_{\mathcal{S}}$ associated to a (locally finite) poset arising from a semigroup $\mathcal{S}$ of $\mathbb{Z}^m$.

In order to do this, we introduce and develop a new approach to study $\mu_{\mathcal{S}}$ by using the Hilbert series associated to $\mathcal{S}$. The latter allows us to provide formulas for $\mu_{\mathcal{S}}$ when $\mathcal{S}$ is a semigroup with unique Betti element, and when $\mathcal{S}$ is a complete intersection numerical semigroup with three generators. We also give a characterization for a locally finite poset to be isomorphic to a semigroup poset. We are thus able to calculate the Möbius function of certain posets (for instance the classical arithmetic Möbius function) by computing the Möbius function of the corresponding semigroup poset.

Joint work with J. Chappelon, L.P. Montejano and I. García-Marco

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Diego Ruano *Aalborg Universitet*

RELATIVE GENERALIZED HAMMING WEIGHTS OF ONE-POINT ALGEBRAIC GEOMETRIC CODES: AN APPLICATION TO SECRET SHARING

Security of linear ramp secret sharing schemes can be characterized by the relative generalized Hamming weights of the involved codes. We devise a method to estimate their value for general one-point algebraic geometric codes which uses information on the corresponding Weierstrass semigroup. As it is demonstrated, for Hermitian codes our bound is often tight.

Alessio Sammartano *Purdue University*

WILF'S CONJECTURE: NEW RESULTS AND NEW QUESTIONS

One of the oldest open problems in the theory of numerical semigroups is Wilf's conjecture, which provides a bound for the Frobenius number in terms of the embedding dimension. We present a new result in support of the conjecture: once the ratio of the multiplicity to the embedding dimension is fixed, the conjecture holds for all numerical semigroups with large enough multiplicity avoiding a finite set of primes. Then we investigate the question of when the equality is achieved, proposing a characterization and providing some evidence in its favor. Finally we consider a possible generalization of the conjecture to the context of one-dimensional local rings. This is a joint work with Alessio Moscariello.

Klara Stokes *University of Skövde*

THE PATTERNS ADMITTED BY A NUMERICAL SEMIGROUP

A pattern admitted by a numerical semigroup S is a multivariate polynomial $p(x_1, \dots, x_n)$ such that $p(s_1, \dots, s_n) \in S$ for all sequences $s_1, \dots, s_n \in S$ with $s_1 \geq \cdots \geq s_n > 0$. Linear patterns appear naturally in numerical semigroups of maximal embedding dimension, in some Weierstrass semigroups and in all numerical semigroups associated with configurations. Previous research in patterns on numerical semigroups has principally focused on the set of numerical semigroups that admit a given pattern. In this talk we will instead study the set of patterns admitted by a given numerical semigroup.

Francesco Strazzanti *Università di Pisa*

NUMERICAL DUPLICATION OF A NUMERICAL SEMIGROUP

If S is a numerical semigroup (briefly n.s.), $b \in S$ is an odd integer, and E is a proper ideal of S, the numerical duplication of S with respect to E and b is defined as the n.s.

$$S \bowtie^b E = 2S \cup (2E + b).$$

This construction arises from commutative algebra. In fact we introduce a family of rings that gives a unified approach to idealization and amalgamated duplication; the numerical duplication arises from a member of this family, if we start with a numerical semigroup ring or with an algebroid branch.
If E is a relative ideal such that $E+E+b \subseteq S$, $S \bowtie^b E$ is still a n.s.; in fact all n.s. can be realized in this way. Using this fact we characterize when a n.s. is almost symmetric. Moreover we generalize some results of Rosales and García-Sánchez about symmetric and pseudo-symmetric n.s. In fact we show that every n.s. of type t is one half of infinitely many almost symmetric n.s. of odd type between 1 and $2t+1$; moreover we show that a n.s. is almost symmetric if and only if it is one half of an almost symmetric n.s. with even type.

References

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Grazia Tamone *Università di Genova*

SYZYGIES AND SMOOTHABILITY OF SOME AS MONOMIAL CURVES

Let X be a monomial algebraic curve in $\mathbb{A}_K^{n+1}$ defined by a numerical semigroup S. The "smoothability" of X , i.e., the existence of at deformations with smooth fibers, assures the "Weierstrass property" of the semigroup S, which is related to the construction of algebraic geometric codes . We deal with semigroups S generated by a generalized arithmetic sequence $m_0, \dots, m_n$ (GS semigroups); in this cases the defining ideal I of the curve X is generated by the 2×2 minors of two matrices . The knowledge of a minimal free resolution of the coordinate ring of X allows us to obtain a "determinantal" description of the first syzygy module . By this tool one can deduce the smoothability of the GS Arf curves; moreover, by suitable deformations of the matrices defining I and via Bertini's theorem, we show the smoothability of the curves X associated to semigroups generated by arithmetic sequences (AS) under the assumption $n \leq 3b$, where $m_0 \equiv b \pmod{n}$. These results and other previous ones in particular ensure the smoothability of all the AS monomial curves with embedding dimension $\nu \leq 7$ and (when $b \neq 2$) $\nu \leq 10$.

Alberto Vigneron-Tenorio *Univerisdad de Cádiz*

CONVEX BODY SEMIGROUPS

Convex body semigroups are generated from compact convex subsets of $\mathbb{R}^k$ with non-empty interior. We show some of their properties and characterize the affine convex body semigroups obtained from circles and polygons of $\mathbb{R}^2$. Besides, these semigroups are used to provide some families of Cohen-Macaulay, Gorenstein and Buchsbaum rings.

Caterina Viola *Univeristà di Catania*

WHEN THE CATENARY AND TAME DEGREE MEET IN EMBEDDING DIMENSION THREE

Let S be an embedding dimension three numerical semigroup. Associated to $n \in S$ define the graph G_n whose vertices are the generators a of S such that $n - a \in S$ and whose edges are the pair of generators ab with $n - (a + b) \in S$. We say that n is a Betti element if G_n is not connected (see for instance [2]).

If S is not symmetric, then it is well known that S has a generic (and unique) minimal presentation (see for instance [2, 3]), and S has three Betti elements. As a consequence of this it was proven in [1] that the tame and catenary degree of S coincide.

If S is symmetric, then it is a complete intersection and consequently the number of $n \in S$ such that its associated graph is not connected is at most two (the number of Betti elements is two).

We show that the catenary degree meets the tame degree if and only if the number of Betti elements of S is one or three (equivalently, $c(S) < t(S)$ if and only if the number of Betti elements of S is two).

This is a joint work with P. A. García-Sánchez and V. Micale.

References

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Keiichi Watanabe *Nihon University*

ULRICH IDEALS IN SEMIGROUP RINGS

I will introduce the notion of Ulrich ideals defined in [1] and discuss about the condition for a semigroup ring to have Ulrich ideals. Also, I will introduce new results by Takahiro Numata.

References

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Santiago Zarzuela *Universitat de Barcelona*

ARITHMETICAL PROPERTIES OF MONOMIAL CURVES OBTAINED BY GLUING

We study arithmetical properties of tangent cones associated to large families of monomial curves obtained by gluing. In particular, we consider the Cohen-Macaulay and Gorenstein properties, and the non-decreasing of the Hilbert functions of monomial curves obtained under a condition that we call specific gluing. The results come from a careful analysis of some special Apéry sets of the numerical semigroups obtained by an specific gluing. In this way we complete and extend the results proved by Arslan et al. in 2009, and also show that for a given monomial curve with a non-decreasing Hilbert function and an integer $q > 1$, all extensions by q, except a finite number, have a non-decreasing Hilbert function.

This is a joint work with Raheleh Jafari.

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