Zernike-Sobolev polynomials and orthogonal expansions on the unit ball

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For $\mu \geqslant 0$, let

$$(f,g)_\mu = f(0)\,g(0) + \lambda \int_{B^2} \nabla f(x) \cdot \nabla g(x)\,(1-||x||^2)^\mu dx, \quad \lambda > 0,$$

be a Sobolev inner product defined on the linear space of polynomials of d variables. Here ∇f is the gradient of f, B^d is the unit ball of $\mathbb{R}^d$ and $||x||$ is the usual Euclidean norm of $x \in \mathbb{R}^d$. In this work, we determine an explicit orthogonal polynomial basis associated with $(\cdot,\cdot)_\mu$ and study approximation properties of Fourier expansions in terms of this basis. In particular, we deduce relations between the partial Fourier sums in terms of the Sobolev polynomials and the partial Fourier sums in terms of the classical ball polynomials. We give an estimate of the approximation error by polynomials of degree at most n in the corresponding Sobolev space.

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References

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