Convergent and asymptotic expansions of Laplace transforms

Chelo Ferreira, José L. López, Pablo Palacios, Pedro Pagola and Ester Pérez Sinusía

[1] *Dpto. de Matemática Aplicada, IUMA, Universidad de Zaragoza, Zaragoza*

e-mail: cferrei@unizar.es, ester.perez@unizar.es

[2] *Dpto. de Ingeniería Matemática e Informática, Universidad Pública de Navarra and INAMAT, Pamplona*

e-mail: jl.lopez@unavarra.es

Abstract

Watson's Lemma provides an asymptotic expansion of Laplace transforms for large values of the transformation parameter z. It is a useful tool in the asymptotic approximation of special functions that have an integral representation in the form of the Laplace transform of a certain function $f(t)$. But in most of the important examples of special functions, the asymptotic expansion derived by means of Watson's Lemma is not convergent. We investigate a modification of Watson's Lemma that transforms the unbounded integration interval $[0,\infty)$ of the Laplace transform into the bounded interval $(0,1]$. Then, we derive an asymptotic expansion of the transformed integral for large z that it is convergent under a mild condition over the function $f(t)$. Moreover, we extend the idea to two dimensions, deriving asymptotic expansions of two-dimensional Laplace transforms for large values of the two transformation parameters that are also convergent. The expansions are accompanied by error bounds. Some examples of special functions are given as illustration, deriving new convergent and asymptotic expansions of these functions.