D2PO-22

Dos Días sobre Polinomios Ortogonales

Granada, 10-11 noviembre 2022

Libro de Resúmenes

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Proyecto de Investigación *Polinomios Ortogonales Multivariados. Aspectos Teóricos y Aplicaciones Científicas (PGC2018-094932-B-I00)*. Ministerio de Ciencia, Innovación y Universidades (MICINN), y el Fondo Europeo de Desarrollo Regional (FEDER).

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Página web

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Lugar

Residencia Universitaria Carmen de Victoria. Universidad de Granada

Horario

Jueves 10 de noviembre

Chairperson: Lidia Fernandez

9:00 - 9:30	Bienvenida
9:30 - 10:15	Francisco Marcellán, *Polinomios ortogonales asociados a truncamientos de la distribución normal.*
10:15 - 11:00	María José Cantero, *Wall polynomials: a tool for Khrushchev's formula.*
11:00 - 11:30	Café.

Chairperson: Miguel Piñar

11:30 - 12:15	Ramón Orive, *From Orthogonal Polynomials to Riesz Equilibrium Problems. The case of unbounded conductors.*
12:15 - 13:00	Chelo Ferreira, *Convergent and asymptotic expansions of Laplace transforms.*
13:00 - 13:45	Amilcar Branqinho, *Applications of quadratic decomposition for bivariate orthogonal polynomials.*
13:45 - 16:00	Almuerzo

Chairperson: Clotilde Martínez

16:00 - 16:45	María Ángeles García Ferrero, *Exceptional Jacobi polynomials.*
16:45 - 17:30	Juan Francisco Mañas, *Asintótica tipo Mehler–Heine para diferentes familias de polinomios ortogonales.*
17:30 - 18:00	Café.

Chairperson: Joaquín Sánchez Lara

18:00 - 18:45	Carmen Escribano, *Autovalores generalizados de matrices de momentos, densidad polinomial y soportes de medidas.*
18:45 - 19:15	Pósteres

Viernes 11 de noviembre

Chairperson: Antonia M. Delgado

09:00 - 09:45	Mirta Castro, *Time-and-band limiting for matrix valued orthogonal polynomials.*
09:45 - 10:30	Joaquím Ortega, *Desigualdades hipercontractivas de polinomios.*
10:30 - 11:00	Café.

Chairperson: Teresa E. Pérez

11:00 - 11:45	Fátima Lizarte, *Sobre la energía logarítmica mínima en la 2-esfera.*
11:45 - 12:30	Misael Marriaga, *Zernike-Sobolev polynomials and orthogonal expansions on the unit ball.*
12:30 - 13:15	Isabel Cação, *A family of hypercomplex orthogonal polynomials and some of their properties.*
13:15 - 13:20	Clausura
13:20 - 15:20	Almuerzo
16:00 - 18:00	Visita Cultural*

* Punto de encuentro: a las 16:00 en la puerta del Palacio de Carlos V
`https://goo.gl/maps/XAZHzxWt2EiXvkRSA`

Pósteres

- Gema Alhama Sales, *The Christoffel function on and in quadratic revolution surfaces.*
- Juan Carlos García Ardila, *Geronimus transformations for sequences of d-orthogonal polynomials.*
- Edmundo Huertas, *On Sobolev-type inner products, matrix factorizations, and higher-order recurrence relations.*
- David Lara, *Constructing Bernstein-Jacobi-type operators with derivation properties.*
- Marlon Recarte, *Simultaneous approximation via Laplacians on the ball.*

Charlas Invitadas

Amílcar Branquinho

Universidade de Coimbra, Portugal

Applications of quadratic decomposition for bivariate orthogonal polynomials.

In this seminar we describe the bivariate polynomial sequences orthogonal to a symmetric weight function in terms of several bivariate polynomial sequences orthogonal with respect to Christoffel transformations of the initial weight under a quadratic transformation. We analyze the construction of a symmetric bivariate orthogonal polynomial sequence from a given one, orthogonal to a weight function defined on the positive plane. In this description plays an important role a sort of Backlund type matrix transformations for the involved three term matrix coefficients. We take as a case study relations between symmetric orthogonal polynomials defined on the ball and on the simplex.

This is a joint work with Teresa Peréz (U Granada) and Ana Foulquié Moreno (U Aveiro).

María José Cantero

Universidad de Zaragoza

Wall polynomials: a tool for Khrushchev's formula.

Khrushchev's formula was introduced in 2001 by Sergei Khrushchev to take advantage of continued fraction methods to study Orthogonal Polynomials on the Unit Circle (OPUC). In this analysis a special role is played by the so-called Wall polynomials. He manages to obtain new and profound results in OPUC theory that surprise the scientific community and revolutionize this theory. The key to reaching these results is the so-called Khrushchev's formula that identifies the Schur function of the orthogonality measure modified by the corresponding OPUC. Curiously, the analogue of Khrushchev's formula for Orthogonal Polynomials on the Real Line (OPRL) was not known until 2018, when it was uncovered by Grünbaum and Velázquez. They obtained the OPRL version in the determinate case using an operator theory point of view. Later, the development of Wall polynomials on the real line, allowed to extend Khrushchev's formula to the indeterminate case. In this talk we present Khrushchev's formula for OPUC and its translation to OPRL, highlighting the peculiarities that such a transfer entails. We also present a simple diagrammatic proof of this formula on the real line which sheds light on its graph theoretical meaning.

This is a joint work with L. Moral, L. Velázquez.

Isabel Cação
University of Aveiro, Portugal

A family of hypercomplex orthogonal polynomials and some of their properties.

A generalization of complex function theory to higher dimensions can be done in the framework of Clifford algebras. An important advantage of this approach compared with the classical generalization by several complex variables is that higher-order differential operators can be factorized in terms of lower-order differential operators. An example is the second-order Laplace operator that admits a factorization in terms of first-order differential operators of Cauchy-Riemann type, in a similar way to the one-dimensional complex case. The holomorphic functions are now null-solutions of a generalized Cauchy-Riemann system and are called monogenic or hyperholomorphic. We present a family of hyperholomorphic polynomials that generalizes the usual holomorphic powers and that are orthogonal with respect to a generalized hypercomplex inner product. They can served as generators of more general hypercomplex polynomials that form building blocks to the construction of an orthogonal basis for the space of n-dimensional hyperholomorphic polynomials. Moreover, the proposed generalized family is formed by Appell polynomials, they satisfy a second-order differential equation and a three-term type recurrence, among other properties.

Mirta M. Castro Smirnova
Universidad de Sevilla

Time-and-band limiting for matrix valued orthogonal polynomials.

In this talk we try to give a survey of the current state of the problem of time-and band-limiting in connection with matrix valued orthogonal polynomials satisfying differential equations (i.e a bispectral situation). For a given family of matrix orthogonal polynomials one considers the global operator defined by a full symmetric matrix or an integral operator, given by the truncated inner products. The problem is to search for a local operator given by a narrow band matrix or a differential operator (respectively), with simple spectrum, commuting with this operator. The existence of a commuting local operator is very useful to compute numerically the eigenfunctions of the given global operator.
This question is motivated by the work of Claude Shannon and a series of papers by D. Slepian, H. Landau and H. Pollak at Bell Labs in the 1960's.

Carmen Escribano
Universidad Politécnica de Madrid

Autovalores generalizados de matrices de momentos, densidad polinomial y soportes de medidas.

En esta charla se presentarán algunos resultados sobre el problema aproximación polinomial en $L^2(\mu)$, para una medida μ en el plano complejo, que han sido obtenidos a través de un enfoque

matricial. Concretamente, el enfoque se ha basado en el análisis del comportamiento asintótico de los autovalores generalizados de las secciones finitas de las matrices de momentos asociadas a una o varias medidas con soporte en el plano complejo.
En primer lugar, se mostrará cómo el comportamiento asintótico de los autovalores mínimos de las secciones finitas de estas matrices repercute en la densidad polinomial. En el caso de medidas cuyo soporte es una curva de Jordan, se presentará una caracterización de la aproximación polinomial en términos de ciertos índices asociados a problemas de optimización matricial.
En esa misma línea, el concepto de autovalor generalizado entre matrices de momentos de dos medidas permitirá comparar, en cierto sentido, las medidas para obtener resultados más generales de aproximación, así como otras propiedades de las medidas en relación a sus soportes.
Finalmente mostraremos algunas aplicaciones al problema de acotación del conjunto de ceros de los polinomios ortogonales de Sobolev.

Los resultados mostrados en esta charla son parte de varios trabajos de investigación en colaboración con Raquel Gonzalo y Emilio Torrano.

Referencias

[1] C. Berg, R. Szwarc. The smallest eigenvalue of Hankel matrices. Constr. Approx., 34: 107-133 (2011).

[2] C. Escribano, R. Gonzalo, E. Torrano. Small eigenvalues of large Hermitian moment matrices. J. Math. Anal. Appl., 374: 470-480 (2011).

[3] C. Escribano, R. Gonzalo, E. Torrano. A characterization of polynomial density on curves via matrix algebra. Mathematics, 7: 1231, (2019).

[4] C. Escribano, R. Gonzalo, E. Torrano. Smallest and largest generalized eigenvalues of large moment matrices and some applications. ArXiv 2203.15453 (2022).

Chelo Ferreira
Universidad de Zaragoza

Convergent and asymptotic expansions of Laplace transforms.

Watson's Lemma provides an asymptotic expansion of Laplace transforms for large values of the transformation parameter z. It is a useful tool in the asymptotic approximation of special functions that have an integral representation in the form of the Laplace transform of a certain function $f(t)$. But in most of the important examples of special functions, the asymptotic expansion derived by means of Watson's Lemma is not convergent. We investigate a modification of Watson's Lemma that transforms the unbounded integration interval $[0, \infty)$ of the Laplace transform into the bounded interval $(0, 1]$. Then, we derive an asymptotic expansion of the transformed integral for large z that it is convergent under a mild condition over the function $f(t)$. Moreover, we extend the idea to two dimensions, deriving asymptotic expansions of two-dimensional Laplace transforms for large values

of the two transformation parameters that are also convergent. The expansions are accompanied by error bounds. Some examples of special functions are given as illustration, deriving new convergent and asymptotic expansions of these functions.

Joint work with José L. López, Pablo Palacios, Pedro Pagola and Ester Pérez Sinusía.

María Ángeles García Ferrero

Basque Center for Applied Mathematics

Exceptional Jacobi polynomials.

Exceptional orthogonal polynomials arise as eigenfunctions of Sturm-Liouville problems and form complete bases in Hilbert spaces despite the fact of missing some degrees. In this talk we will focus on those which generalize the classical Jacobi polynomials. We will see how we can construct exceptional Jacobi polynomials from the classical ones beyond the standard method. We will also discuss different properties which can be expected depending on the parameters of the original family.

This is a joint work with D. Gómez-Ullate and R. Milson.

Fátima Lizarte

Universidad de Cantabria

Sobre la energía logarítmica mínima en la 2-esfera.

Smale, ganador de la medalla Fields en 1966, elaboró una lista de 18 problemas a finales del siglo XX, reuniendo algunos de los principales retos matemáticos para el siglo XXI. El problema número 7 es dar una descripción simple y eficiente, o alternativamente describir un algoritmo, para colocar N puntos en la 2-esfera tal que su potencial logarítmico esté muy cerca del mínimo. Una dificultad importante en este problema es que el valor de la energía logarítmica mínima de N puntos en la esfera no se conoce completamente. Su conocimiento actual es:

$$\kappa N^2 - \frac{1}{2} N \ln N + C_{\log} N + o(N),$$

siendo κ la energía continua y $C_{\log}$ una constante tal que

$$-0{,}0568\ldots = \ln 2 - \frac{3}{4} \leqslant C_{\log} \leqslant 2\ln 2 + \frac{1}{2}\ln\frac{2}{3} + 3\ln\frac{\sqrt{\pi}}{\Gamma(1/3)} = -0{,}0556\ldots,$$

donde la cota inferior ha sido probada por Lauritsen usando resultados sofisticados. De hecho, la cota superior ha sido conjeturada a ser una igualdad y es uno de los problemas abiertos más importantes del área. En esta charla, presentaré una prueba alternativa para la cota inferior de $C_{\log}$, llegando al mismo valor mediante un cálculo directo. También mostraré cómo este nuevo enfoque se puede generalizar para obtener cotas inferiores para la energía de Green en $\mathbb{S}^n$.
Esto es un trabajo conjunto con Carlos Beltrán.

Juan Francisco Mañas
Universidad de Almería

Asintótica tipo Mehler–Heine para diferentes familias de polinomios ortogonales.

En esta charla haremos un breve repaso histórico por los resultados sobre asintótica tipo Mehler–Heine para diferentes familias de polinomios ortogonales y polinomios hipergeométricos. Este tipo de asintótica tiene como principal propiedad establecer una relación límite entre los polinomios con un reescalamiento adecuado de la variable y funciones de Bessel de primera especie. A continuación, mostraremos este tipo de asintótica local para una familia de polinomios q-hipergeométricos y, como consecuencia, obtendremos las fórmulas tipo Mehler–Heine para algunas familias de q-polinomios ortogonales. Finalmente, usando el teorema de Hurwitz, se puede establecer una interesante relación entre los ceros reescalados de los polinomios q-hipergeométricos y los ceros de funciones de q-Bessel de primera especie (para más detalles ver [1]).

Este es un trabajo conjunto con Juan J. Moreno Balcázar.

Referencias

[1] J.F. Mañas, J.J. Moreno-Balcázar, Asymptotics for some q-hypergeometric polynomials Results, Math. Accepted

Francisco Marcellán
Universidad Carlos III de Madrid

Polinomios ortogonales asociados a truncamientos de la distribución normal.

En esta presentación estudiaremos propiedades analíticas de polinomios ortogonales respecto a medidas definidas por truncamiento de la distribución normal. En particular, consideraremos el caso en que el soporte es un intervalo simétrico $(-t, t)$ (los llamados polinomios de Rys). Son polinomios semiclásicos y simétricos de los que analizaremos la relación de recurrencia a tres términos, con estimaciones de sus parámetros (que dependen de t), los operadores de creación y aniquilación asociados y deduciremos la correspondiente ecuación holonómica que satisfacen dichos polinomios de manera que surge de manera natural una interpretación electrostática de sus ceros. Asimismo estudiaremos su comportamiento dinámico en función de t y establecemos la conexión con una perturbación de Toda de la medida de Lebesgue en el intervalo $(-1, 1)$. Este tipo de polinomios aparece en Química Cuántica y el estudio de sus ceros para abordar de manera eficiente las correspondientes fórmulas de cuadratura Gaussiana ha sido objeto de atención durante los últimos años.

Este es trabajo conjunto con Diego Dominici, Department of Mathematics, State University at New Paltz.

Misael E. Marriaga
Universidad Rey Juan Carlos

Zernike-Sobolev polynomials and orthogonal expansions on the unit ball.

For $\mu \geq 0$, let

$$(f,g)_\mu = f(0)g(0) + \lambda \int_{B^d} \nabla f(x) \cdot \nabla g(x)(1-\|x\|^2)^\mu dx\,, \qquad \lambda > 0,$$

be a Sobolev inner product deined on the linear space of polynomials of d variables. Here ∇f is the gradient of f, B^d is the unit ball of $\mathbb{R}^d$ and $\|x\|$ is the usual Euclidean norm of $x \in \mathbb{R}^d$. In this work, we determine an explicit orthogonal polynomial basis associated with $(\cdot,\cdot)_\mu$ and study approximation properties of Fourier expansions in terms of this basis. In particular, we deduce relations between the partial Fourier sums in terms of the Sobolev polynomials and the partial Fourier sums in terms of the classical ball polynomials. We give an estimate of the approximation error by polynomials of degree at most n in the corresponding Sobolev space.

Joint work with: Marlon J. Recarte, Teresa E. Pérez, Miguel A. Piñar.

Referencias

[1] Dai F., Xu Y., Approximation theory and harmonic analysis on spheres and balls, Springer Monographs in Mathematics, Springer, New York, 2013.

[2] Dunkl C.F., Xu Y., Orthogonal polynomials of several variables, Encyclopedia of Mathematics and its Applications, Vol. 155, 2nd ed., Cambridge University Press, Cambridge, 2014.

[3] Piñar M. A., Xu Y., Best polynomial approximation on the unit ball, IMA Journal of Numerical Analysis, Vol. 38, Issue 3 (2018), 1209-1228.

Ramón Orive
Universidad de La Laguna

From Orthogonal Polynomials to Riesz Equilibrium Problems. The case of unbounded conductors.

In our case, the study of the asymptotic behavior of orthogonal polynomials led us to delve into the study of equilibrium problems for logarithmic potentials and, in turn, for general Riesz potentials. In this talk, after briefly reviewing these connections, we shall focus in the case of Riesz equilibrium problems for unbounded conductors. We shall present some recent results about existence and compactness of the equilibrium measure.

This talk is based on a recent work with P. Dragnev (PFW, IN, USA), E. B. Saff (Vanderbilt Univ., Nasville, TEN, USA) and F. Wielonsky (Univ. Aix-Marseille, France).

Joaquim Ortega Cerdà
Universidad de Barcelona

Desigualdades hipercontractivas de polinomios.

Si dotamos los polinomios de grado n con la norma de Bombieri tenemos un espacio de Hilbert con núcleo reproductor. Demostraremos que todo funcional convexo de la norma del espacio alcanza su extremo en el núcleo reproductor. Esto permite dar una nueva demostración muy elemental de una conjetura física sobre el comportamiento de la entropía de Wherl para estados coherentes de Bloch que fue demostrada inicialmente por Lieb y Solowej. Asimismo obtendremos resultados análogos a la desigualdad de Faber-Krhan en el contexto de los polinomios.

Este es un trabajo conjunto con N. Fabio, A. Kulikov y P. Tilli.

Pósteres

Gema Alhama Sales

Universidad de Granada

The Christoffel function on and in quadratic revolution surfaces.

In [1], some explicit constructions of orthogonal polynomials inside quadratic bodies of revolution, including cones, hyperboloids, and paraboloids, were given. Also, orthogonal polynomials on the surface of quadratic solids of revolution were constructed from spherical harmonics. In both cases, the corresponding kernels can be represented in terms of the kernels associated to classical weight functions on the line. From this representation and some well known results by Totik [2], we obtain asymptotics estimates of the Christoffel functions for measures supported on and in quadratic revolution surfaces.

This is a joint work with Miguel Piñar.

Referencias

[1] Sheehan Olver and Yuan Xu. Orthogonal polynomials in and on a quadratic Surface of revolution.

[2] Vilmos Totik. Asymptotics for Christoffel functions with varying weights. Adv. in Appl. Math. 25, no. 4, (2000) 322–351.

Juan Carlos García Ardila

Universidad Politécnica de Madrid

Geronimus transformations for sequences of d-orthogonal polynomials.

In this work an extension of the concept of Geronimus transformation for sequences of d-orthogonal polynomials $\{P_n\}$ is introduced. The transformed sequences $\{P_n^{(k)}\}$, for $k = 1, \ldots, d$, are analyzed and some relationships between these new sequences of polynomials are given. Also the associated Hessenberg matrices and their transformed are studied.

Edmundo Huertas
Universidad de Alcalá

On Sobolev-type inner products, matrix factorizations, and higher-order recurrence relations.

It is well known that Sobolev-type orthogonal polynomials with respect to measures supported on the real line satisfy higher-order recurrence relations and these can be expressed as a (2N+1)-banded symmetric semi-infinite matrix. In this work we state the connection between these (2N+1)-banded matrices and the Jacobi matrices associated with the three-term recurrence relation satisfied by the standard sequence of orthonormal polynomials with respect to the 2-iterated Christoffel transformation of the measure. This is a joint work with C. Hermoso, A. Lastra, and F. Marcellán.

This is a joint work with Alberto Lastra.

David Lara
Universidad de Granada

Constructing Bernstein-Jacobi-type operators with derivation properties.

Bernstein polynomials were introduced by S. Bernstein in 1912 to provide a constructive proof of the Weierstrass approximation theorem. In this way it was established that every continuous function defined in the interval $[0, 1]$ can be uniformly approximated by Bernstein polynomials in such interval.

In this work we study a modification of the Bernstein operator that was studied in [2] by means of the Jacobi inner product. We analyze its properties on different types of functions and their possible applications.

This is a joint work with Teresa E. Pérez.

Referencias

[1] T. S. Chihara, *An Introduction to Orthogonal Polynomials*, Mathematics and its Applications, vol. 13, Gordon and Breach, New York, 1978.

[2] V. Gupta, A. J. López-Moreno, J. M. Latorre-Palacios, *On simultaneous approximation of the Bernstein Durrmeyer operators*, Appl. Math. Comput. 213 (2009), no. 1, 112–120.

[3] G. G. Lorentz, *Bernstein Polynomials*, Second edition. Chelsea Publishing Co., New York, 1986.

[4] P. Sablonnière, *Opérateurs de Bernstein-Jacobi et Polynômes Orthogonaux*, Publ. ANO 37, Laboratoire de Calcul, Lille, 1981.

Marlon Recarte
Universidad Nacional Autónoma de Honduras

Simultaneous approximation via Laplacians on the ball.

In this work, we study the orthogonal structure on the unit ball of $\mathbb{R}^d$ with respect to the Sobolev inner product

$$\begin{aligned}\langle f, g\rangle_{\Delta} = & \frac{1}{\sigma_{d-1}} \int_{\mathbf{S}^{d-1}} f(\xi)\, g(\xi)\, d\sigma(\xi) \\ & + \frac{\lambda}{8\,\sigma_{d-1}} \int_{\mathbf{B}^d} \Delta[(1-\|x\|^2) f(x)]\, \Delta[(1-\|x\|^2) g(x)]\, dx, \quad \lambda > 0.\end{aligned}$$

Our main contribution consists in the study of the approximation properties of the Fourier sums with respect to orthogonal polynomials associated with $\langle \cdot, \cdot \rangle_{\Delta}$. In particular, we estimate the error of simultaneous approximation of a function, its partial derivatives, and its Laplacian in the $L^2(\mathbf{B}^d)$ space.

Participantes

1. Manuel Alfaro. Universidad de Zaragoza.
2. Gema Alhama Salés (U.Granada).
3. Amílcar Branquinho (U. Coimbra, Portugal).
4. Cleonice Bracciali (U. Estadual Paulista, Brasil).
5. Isabel Cação (U. Aveiro, Portugal).
6. María José Cantero (U. Zaragoza).
7. Mirta Castro (U. Sevilla).
8. Antonia M. Delgado (U. Granada).
9. Carmen Escribano (U. Politécnica de Madrid).
10. Lidia Fernández (U. Granada).
11. Chelo Ferreira González (U. Zaragoza).
12. Ana Foulquié (U. Aveiro, Portugal).
13. Juan Carlos García Ardila (U. Politécnica de Madrid).
14. María Ángeles García Ferrero (Basque Center for Applied Mathematics).
15. Edmundo Huertas (U. Alcalá).
16. Alberto Lastra (U. Alcalá).
17. David Lara Velasco (U. Granada).
18. Fátima Lizarte (U. Cantabria).
19. Juan Francisco Mañas Mañas (U. Almería).
20. Francisco Marcellán Español (U. Carlos III de Madrid).
21. Misael Marriaga Castillo (U. Rey Juan Carlos).
22. Clotilde Martínez Álvarez (U. Granada).
23. Dieudonne Mbouna (U. Almería).
24. Juanjo Moreno-Balcázar (U. Almería).
25. Ruth Oliva Zuniga (U. Nacional Autónoma de Honduras, Honduras).
26. Ramón Orive Rodríguez (U. La Laguna).

27. Joaquim Ortega Cerdà (U. Barcelona).
28. Teresa E. Pérez (U. Granada).
29. Miguel A. Piñar (U. Granada).
30. Marlon Recarte Castellanos (U. Nacional Autónoma de Honduras, Honduras).
31. Miguel Luis Rodríguez (U. Granada).
32. José Luis Ruiz Benito (U. Granada).
33. Joaquín Sánchez Lara (U. Granada).
34. Juan Antonio Villegas Recio (U. Granada).
35. Alejandro Zarzo Altarejos (U. Politécnica de Madrid).