Complementary Romanovski-Routh polynomials

A. Sri Ranga

DMAp/IBILCE, UNESP - Universidade Estadual Paulista

For \( b = \lambda + i\eta, \lambda > 0 \), the complementary Romanovski-Routh polynomials \( P_n(b; x) \) can be given by the hypergeometric expression

\[
P_n(b; x) = \frac{(x - i)^n}{2^n} \frac{(2\lambda)_n}{(\lambda)_n} {}_2F_1\left(-n, b; b + \frac{2i}{x - i}\right), \quad n \geq 1.
\]

They satisfy the three term recurrence

\[
P_{n+1}(b; x) = (x - c_n^{(b)})P_n(b; x) - d_n^{(b)}(x^2 + 1)P_{n-1}(b; x), \quad n \geq 1,
\]

with \( P_0(b; x) = 1 \) and \( P_1(b; x) = x - c_1^{(b)} \), where

\[
c_n^{(b)} = \frac{\eta}{\lambda + n - 1} \quad \text{and} \quad d_n^{(b)} = d_n^{(\lambda)} = \frac{1}{4} \frac{n(2\lambda + n - 1)}{(\lambda + n - 1)(\lambda + n)}, \quad n \geq 1.
\]

Moreover, if \( \lambda > 1/2 \) then they also satisfy the varying orthogonality

\[
\int_{-\infty}^{\infty} x^m P_n(b; x) \frac{1}{(1 + x^2)^n} \frac{e^{-\arccot x}}{(1 + x^2)^\lambda} dx = \gamma_{n}^{(\lambda)} \delta_{m,n}, \quad m = 0, 1, \ldots, n.
\]

In this talk we will look at some recent developments with respect these polynomials.