Serial coalgebras and their valued Gabriel quivers

Gabriel Navarro

Departamento de Álgebra
Universidad de Granada

Noncommutative Rings and Geometry

Almería, September 2007

Joint work with **José Gómez Torrecillas**

Serial coalgebras and their valued Gabriel quivers

arXiv:0707.0132v1 [math.RT]

Continuing the papers:

- J. **Cuadra** and J. **Gómez-Torrecillas**, *Serial coalgebras, J. Pure Appl. Algebra 189 (2004), 89–107.*
- *J. **Kosakowska** and D. **Simson**, Hereditary coalgebras and representations of species, J. Algebra 293 (2005), 457–505.*

Joint work with **José Gómez Torrecillas**

Serial coalgebras and their valued Gabriel quivers

arXiv:0707.0132v1 [math.RT]

Continuing the papers:

- J. **Cuadra** and J. **Gómez-Torrecillas**, *Serial coalgebras, J. Pure Appl. Algebra 189 (2004), 89–107.*
- *J. **Kosakowska** and D. **Simson**, Hereditary coalgebras and representations of species, J. Algebra 293 (2005), 457–505.*

What are we trying to do?

Main aim

Study serial coalgebras by means of their valued Gabriel quiver

over an arbitrary field!!!

Looking for analogies with f. d. algebras:

- shape of the quiver
- description of the comodules (A-R quiver)
- are all comodules a direct sum of uniserials?
- Eisenbud-Griffith theorem?
- does localization preserve seriality?

What are we trying to do?

Main aim

Study serial coalgebras by means of their valued Gabriel quiver

over an arbitrary field!!!

Looking for analogies with f. d. algebras:

- shape of the quiver
- description of the comodules (A-R quiver)
- are all comodules a direct sum of uniserials?
- Eisenbud-Griffith theorem?
- does localization preserve seriality?

What are we trying to do?

Main aim

Study serial coalgebras by means of their valued Gabriel quiver

over an arbitrary field!!!

Looking for analogies with f. d. algebras:

- shape of the quiver
- description of the comodules (A-R quiver)
- are all comodules a direct sum of uniserials?
- Eisenbud-Griffith theorem?
- does localization preserve seriality?

What are we trying to do?

Main aim

Study serial coalgebras by means of their valued Gabriel quiver

over an arbitrary field!!!

Looking for analogies with f. d. algebras:

- shape of the quiver
- description of the comodules (A-R quiver)
- are all comodules a direct sum of uniserials?
- Eisenbud-Griffith theorem?
- does localization preserve seriality?

What are we trying to do?

Main aim

Study serial coalgebras by means of their valued Gabriel quiver

over an arbitrary field!!!

Looking for analogies with f. d. algebras:

- shape of the quiver
- description of the comodules (A-R quiver)
- are all comodules a direct sum of uniserials?
- Eisenbud-Griffith theorem?
- does localization preserve seriality?

What are we trying to do?

Main aim

Study serial coalgebras by means of their valued Gabriel quiver

over an arbitrary field!!!

Looking for analogies with f. d. algebras:

- shape of the quiver
- description of the comodules (A-R quiver)
- are all comodules a direct sum of uniserials?
- Eisenbud-Griffith theorem?
- does localization preserve seriality?

Definitions: serial coalgebra

Socle filtration: $\mathrm{Soc}\ M \subset \mathrm{Soc}^2 M \subset \mathrm{Soc}^3 M \subset \cdots \subset M$

- $\mathrm{Soc}\ M$ is the socle of M
- $\dfrac{\mathrm{Soc}^n M}{\mathrm{Soc}^{n-1} M} = \mathrm{Soc}\left(\dfrac{M}{\mathrm{Soc}^{n-1} M}\right)$.

Definition

M is uniserial if $\mathrm{Soc} M \subset \mathrm{Soc}^2 M \subset \cdots \subset M$ is a composition series

Definition

C is (right, left) serial if (right, left) indecomposable injectives are uniserial.

Definitions: serial coalgebra

Socle filtration: $\mathrm{Soc}\ M \subset \mathrm{Soc}^2 M \subset \mathrm{Soc}^3 M \subset \cdots \subset M$

- $\mathrm{Soc}\ M$ is the socle of M
- $\dfrac{\mathrm{Soc}^n M}{\mathrm{Soc}^{n-1} M} = \mathrm{Soc}\left(\dfrac{M}{\mathrm{Soc}^{n-1} M}\right)$.

Definition

M is uniserial if $\mathrm{Soc} M \subset \mathrm{Soc}^2 M \subset \cdots \subset M$ is a composition series

Definition

C is (right, left) serial if (right, left) indecomposable injectives are uniserial.

Definitions: serial coalgebra

Socle filtration: $\mathrm{Soc}\ M \subset \mathrm{Soc}^2 M \subset \mathrm{Soc}^3 M \subset \cdots \subset M$

- $\mathrm{Soc}\ M$ is the socle of M
- $\dfrac{\mathrm{Soc}^n M}{\mathrm{Soc}^{n-1} M} = \mathrm{Soc}\left(\dfrac{M}{\mathrm{Soc}^{n-1} M}\right)$.

Definition

M is uniserial if $\mathrm{Soc} M \subset \mathrm{Soc}^2 M \subset \cdots \subset M$ is a composition series

Definition

C is (right, left) serial if (right, left) indecomposable injectives are uniserial.

Definitions: valued Gabriel quiver

C basic coalgebra over an arbitrary field
$\{S_i\}_{i \in I_C}$ set of simple C-comodules

Definition

The (right) valued Gabriel quiver of C is:

- Vertices are simple comodules
- There exists $S_1 \xrightarrow{(a,b)} S_2$ if and only if
  - $\mathrm{Ext}^1_C(S_1, S_2) \neq 0$
  - $a = \dim_{\mathrm{End}_C(S_1)} \mathrm{Ext}^1_C(S_1, S_2)$
  - $b = \dim_{\mathrm{End}_C(S_2)} \mathrm{Ext}^1_C(S_1, S_2)$

C basic coalgebra over an arbitrary field
$\{S_i\}_{i \in I_C}$ set of simple C-comodules

Definition

The (right) valued Gabriel quiver of C is:

- Vertices are simple comodules
- There exists $S_1 \xrightarrow{(a,b)} S_2$ if and only if
  - $\mathrm{Ext}^1_C(S_1, S_2) \neq 0$
  - $a = \dim_{\mathrm{End}_C(S_1)} \mathrm{Ext}^1_C(S_1, S_2)$
  - $b = \dim_{\mathrm{End}_C(S_2)} \mathrm{Ext}^1_C(S_1, S_2)$

Definitions: valued Gabriel quiver

C basic coalgebra over an arbitrary field
$\{S_i\}_{i\in I_C}$ set of simple C-comodules

Definition

The (right) valued Gabriel quiver of C is:

- Vertices are simple comodules
- There exists $S_1 \xrightarrow{(a,b)} S_2$ if and only if
  - $\mathrm{Ext}^1_C(S_1, S_2) \neq 0$
  - $a = \dim_{\mathrm{End}_C(S_1)} \mathrm{Ext}^1_C(S_1, S_2)$
  - $b = \dim_{\mathrm{End}_C(S_2)} \mathrm{Ext}^1_C(S_1, S_2)$

Definitions: valued Gabriel quiver

C basic coalgebra over an arbitrary field
$\{S_i\}_{i \in I_C}$ set of simple C-comodules

Definition

The (right) valued Gabriel quiver of C is:

- Vertices are simple comodules
- There exists $S_1 \xrightarrow{(a,b)} S_2$ if and only if
  - $\mathrm{Ext}^1_C(S_1, S_2) \neq 0$
  - $a = \dim_{\mathrm{End}_C(S_1)} \mathrm{Ext}^1_C(S_1, S_2)$
  - $b = \dim_{\mathrm{End}_C(S_2)} \mathrm{Ext}^1_C(S_1, S_2)$

C basic coalgebra over an arbitrary field
$\{S_i\}_{i \in I_C}$ set of simple C-comodules

Definition

The (right) valued Gabriel quiver of C is:

- Vertices are simple comodules
- There exists $S_1 \xrightarrow{(a,b)} S_2$ if and only if
  - $\mathrm{Ext}^1_C(S_1, S_2) \neq 0$
  - $a = \dim_{\mathrm{End}_C(S_1)} \mathrm{Ext}^1_C(S_1, S_2)$
  - $b = \dim_{\mathrm{End}_C(S_2)} \mathrm{Ext}^1_C(S_1, S_2)$

Properties of the valued Gabriel quiver

Proposition

The immediate predecessors of S_i are given by $\dfrac{\mathrm{Soc}^2 E_i}{\mathrm{Soc}\, E_i}$.

Corollary

C is right serial if and only if:

- *Each vertex is the target of at most one arrow.*
- *The arrows are labeled by $(1, d)$.*

Proposition

The left valued Gabriel quiver is the opposite to the right valued Gabriel quiver.

Properties of the valued Gabriel quiver

Proposition

The immediate predecessors of S_i are given by $\frac{\mathrm{Soc}^{\,2} E_i}{\mathrm{Soc}\, E_i}$.

Corollary

C is right serial if and only if:

- *Each vertex is the target of at most one arrow.*
- *The arrows are labeled by $(1, d)$.*

Proposition

The left valued Gabriel quiver is the opposite to the right valued Gabriel quiver.

Properties of the valued Gabriel quiver

Proposition

The immediate predecessors of S_i are given by $\dfrac{\mathrm{Soc}^{\,2} E_i}{\mathrm{Soc}\, E_i}$.

Corollary

C is right serial if and only if:

- *Each vertex is the target of at most one arrow.*
- *The arrows are labeled by* $(1, d)$.

Proposition

The left valued Gabriel quiver is the opposite to the right valued Gabriel quiver.

Theorem

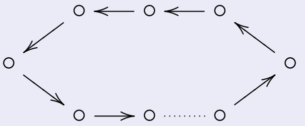
*C is **serial** if and only if its valued Gabriel quiver is*

(a) $_\infty\mathbb{A}_\infty$: $\cdots\cdots\cdots \circ \longrightarrow \circ \longrightarrow \circ \longrightarrow \circ \longrightarrow \circ \cdots\cdots\cdots$

(b) $\mathbb{A}_\infty$: $\circ \longrightarrow \circ \longrightarrow \circ \longrightarrow \circ \longrightarrow \circ \longrightarrow \circ \cdots\cdots\cdots$

(c) $_\infty\mathbb{A}$: $\cdots\cdots\cdots \circ \longrightarrow \circ \longrightarrow \circ \longrightarrow \circ \longrightarrow \circ \longrightarrow \circ$

(d) $\mathbb{A}_n$: $\circ \longrightarrow \circ \longrightarrow \circ \cdots\cdots\cdots \circ \longrightarrow \circ \longrightarrow \circ$ $\quad n \geq 1$

(e) $\widetilde{\mathbb{A}}_n$:  $\quad n \geq 1$

and the labels are $(1,1)$.

Characterization by means of quivers

Theorem

*C is **serial** if and only if its valued Gabriel quiver is*

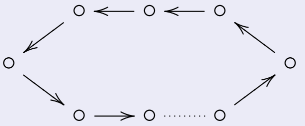
(a) ${}_{\infty}\mathbb{A}_{\infty}$: $\cdots\cdots \circ \longrightarrow \circ \longrightarrow \circ \longrightarrow \circ \longrightarrow \circ \cdots\cdots$

(b) $\mathbb{A}_{\infty}$: $\circ \longrightarrow \circ \longrightarrow \circ \longrightarrow \circ \longrightarrow \circ \longrightarrow \circ \cdots\cdots$

(c) ${}_{\infty}\mathbb{A}$: $\cdots\cdots \circ \longrightarrow \circ \longrightarrow \circ \longrightarrow \circ \longrightarrow \circ \longrightarrow \circ$

(d) $\mathbb{A}_n$: $\circ \longrightarrow \circ \longrightarrow \circ \cdots\cdots \circ \longrightarrow \circ \longrightarrow \circ$ $\quad n \geq 1$

(e) $\widetilde{\mathbb{A}}_n$:



$n \geq 1$

and the labels are $(1, 1)$.

Finite dimensional comodules

Proposition

TFAE:

- *M is uniserial.*
- *Every f.d. subcomodule of M is uniserial.*

Theorem

C serial coalgebra, M indecomposable f.d. comodule

- *$M \cong \mathrm{Soc}^n E_i = S_i^n$, E_i indecomposable injective.*
- *C has almost split sequence, in particular,*

$$\mathrm{Soc}^n E \xrightarrow{\binom{i}{p}} \mathrm{Soc}^{n+1} E \oplus \frac{\mathrm{Soc}^n E}{\mathrm{Soc}\ E} \xrightarrow{(q\ -j)} \frac{\mathrm{Soc}^{n+1} E}{\mathrm{Soc}\ E}$$

is almost split.

Finite dimensional comodules

Proposition

TFAE:

- *M is uniserial.*
- *Every f.d. subcomodule of M is uniserial.*

Theorem

C serial coalgebra, M indecomposable f.d. comodule

- *$M \cong \mathrm{Soc}^n E_i = S_i^n$, E_i indecomposable injective.*
- *C has almost split sequence, in particular,*

$$\mathrm{Soc}^n E \xrightarrow{\binom{i}{p}} \mathrm{Soc}^{n+1} E \oplus \frac{\mathrm{Soc}^n E}{\mathrm{Soc}\, E} \xrightarrow{(q\,-j)} \frac{\mathrm{Soc}^{n+1} E}{\mathrm{Soc}\, E}$$

is almost split.

Finite dimensional comodules

Proposition

TFAE:

- *M is uniserial.*
- *Every f.d. subcomodule of M is uniserial.*

Theorem

C serial coalgebra, M indecomposable f.d. comodule

- *$M \cong \mathrm{Soc}^n E_i = S_i^n$, E_i indecomposable injective.*
- *C has almost split sequence, in particular,*

$$\mathrm{Soc}^n E \xrightarrow{\binom{i}{p}} \mathrm{Soc}^{n+1} E \oplus \frac{\mathrm{Soc}^n E}{\mathrm{Soc}\, E} \xrightarrow{(q\ -j)} \frac{\mathrm{Soc}^{n+1} E}{\mathrm{Soc}\, E}$$

is almost split.

Proposition

TFAE:

- *M is uniserial.*
- *Every f.d. subcomodule of M is uniserial.*

Theorem

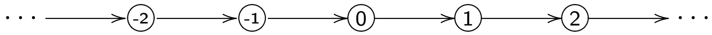
C serial coalgebra, M indecomposable f.d. comodule

- *$M \cong \mathrm{Soc}^n E_i = S_i^n$, E_i indecomposable injective.*
- *C has almost split sequence, in particular,*

$$\mathrm{Soc}^n E \xrightarrow{\binom{i}{p}} \mathrm{Soc}^{n+1} E \oplus \frac{\mathrm{Soc}^n E}{\mathrm{Soc}\ E} \xrightarrow{(q\ -j)} \frac{\mathrm{Soc}^{n+1} E}{\mathrm{Soc}\ E}$$

is almost split.

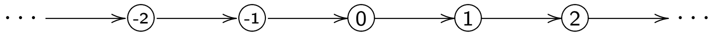
Q



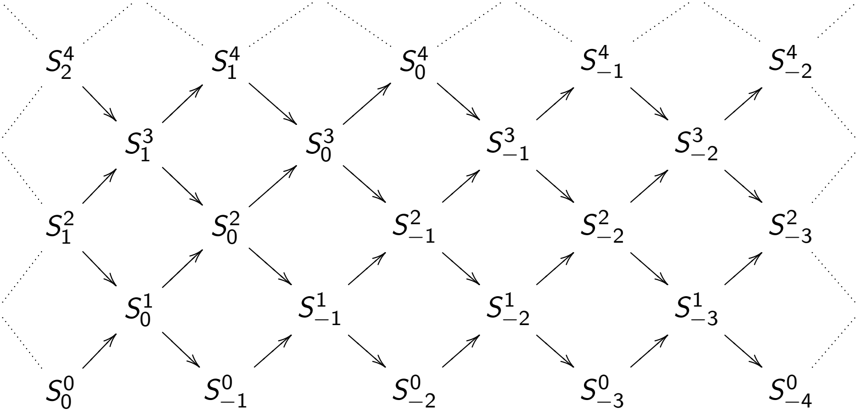
· · ·
-2
-1
0
1
2
· · ·

S^4_2 S^4_1 S^4_0 S^4_{-1} S^4_{-2}
S^3_1 S^3_0 S^3_{-1} S^3_{-2}
S^2_1 S^2_0 S^2_{-1} S^2_{-2} S^2_{-3}
S^1_0 S^1_{-1} S^1_{-2} S^1_{-3}
S^0_0 S^0_{-1} S^0_{-2} S^0_{-3} S^0_{-4}

Q

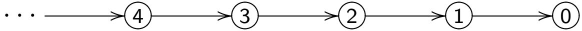


-2
-1
0
1
2



S_2^4
S_1^4
S_0^4
S_{-1}^4
S_{-2}^4
S_1^3
S_0^3
S_{-1}^3
S_{-2}^3
S_1^2
S_0^2
S_{-1}^2
S_{-2}^2
S_{-3}^2
S_0^1
S_{-1}^1
S_{-2}^1
S_{-3}^1
S_0^0
S_{-1}^0
S_{-2}^0
S_{-3}^0
S_{-4}^0

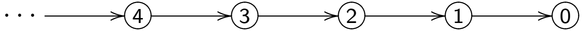
Q



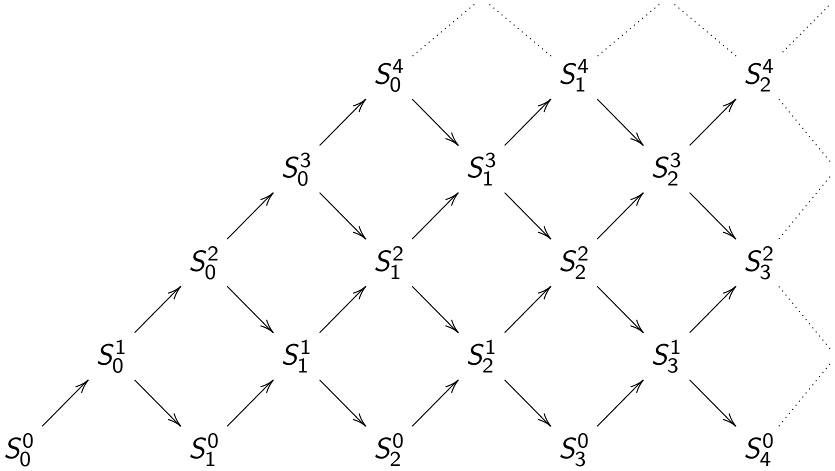
4
3
2
1
0

S_0^4 S_1^4 S_2^4
S_0^3 S_1^3 S_2^3
S_0^2 S_1^2 S_2^2 S_3^2
S_0^1 S_1^1 S_2^1 S_3^1
S_0^0 S_1^0 S_2^0 S_3^0 S_4^0

Q

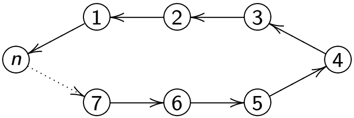


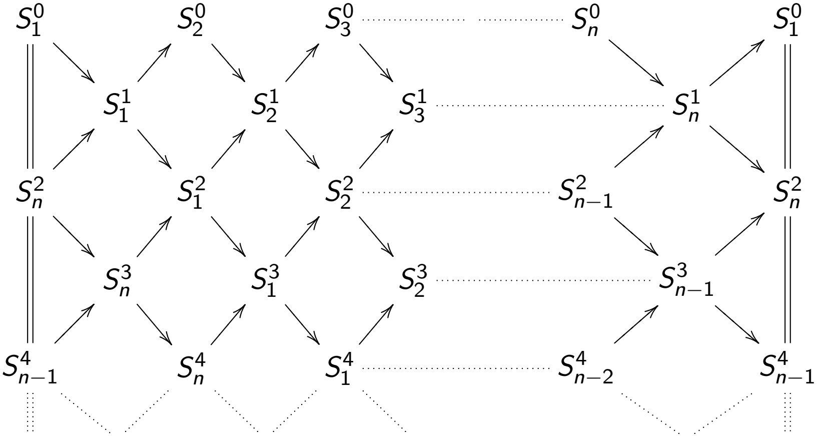
· · ·
4
3
2
1
0



S_0^4
S_1^4
S_2^4
S_0^3
S_1^3
S_2^3
S_0^2
S_1^2
S_2^2
S_3^2
S_0^1
S_1^1
S_2^1
S_3^1
S_0^0
S_1^0
S_2^0
S_3^0
S_4^0

Q


1
2
3
4
5
6
7
n


S_1^0 S_2^0 S_3^0 S_n^0 S_1^0
S_1^1 S_2^1 S_3^1 S_n^1
S_n^2 S_1^2 S_2^2 S_{n-1}^2 S_n^2
S_n^3 S_1^3 S_2^3 S_{n-1}^3
S_{n-1}^4 S_n^4 S_1^4 S_{n-2}^4 S_{n-1}^4

Infinite dimensional comodules

Theorem

A serial finite dimensional algebra, then each module is a direct sum of uniserial modules

And for coalgebras?? **Not really!!**

Proposition

*C serial. Each comodule is **direct sum of uniserial comudules** if and only if C is **pure-semisimple**.*

Counterexample

Consider the path coalgebra of

Theorem

A serial finite dimensional algebra, then each module is a direct sum of uniserial modules

And for coalgebras?? **Not really!!**

Proposition

C serial. Each comodule is ***direct sum of uniserial comudules*** *if and only if C is* ***pure-semisimple****.*

Counterexample

Consider the path coalgebra of

Infinite dimensional comodules

Theorem

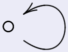
A serial finite dimensional algebra, then each module is a direct sum of uniserial modules

And for coalgebras?? **Not really!!**

Proposition

*C serial. Each comodule is **direct sum of uniserial comudules** if and only if C is **pure-semisimple**.*

Counterexample

Consider the path coalgebra of 

$\mathcal{A}$ dense subcategory of an abelian category $\mathcal{C}$:

- There is a **quotient functor** $T : \mathcal{C} \to \mathcal{C}/\mathcal{A}$.
- $\mathcal{A}$ is **localizing** if T has a right adjoint functor $S : \mathcal{C}/\mathcal{A} \to \mathcal{C}$ (**section functor**).
- $\mathcal{A}$ is **perfect localizing** if S is exact.

$$\mathcal{C} \underset{S}{\overset{T}{\rightleftarrows}} \mathcal{C}/\mathcal{A}$$

Proposition

- *T is an exact functor.*
- *S is a fully faithful and left exact functor.*

$\mathcal{A}$ dense subcategory of an abelian category $\mathcal{C}$:

- There is a **quotient functor** $T : \mathcal{C} \to \mathcal{C}/\mathcal{A}$.
- $\mathcal{A}$ is **localizing** if T has a right adjoint functor $S : \mathcal{C}/\mathcal{A} \to \mathcal{C}$ (**section functor**).
- $\mathcal{A}$ is **perfect localizing** if S is exact.

$$\mathcal{C} \underset{S}{\overset{T}{\rightleftarrows}} \mathcal{C}/\mathcal{A}$$

Proposition

- *T is an exact functor.*
- *S is a fully faithful and left exact functor.*

$\mathcal{A}$ dense subcategory of an abelian category $\mathcal{C}$:

- There is a **quotient functor** $T : \mathcal{C} \to \mathcal{C}/\mathcal{A}$.
- $\mathcal{A}$ is **localizing** if T has a right adjoint functor $S : \mathcal{C}/\mathcal{A} \to \mathcal{C}$ (**section functor**).
- $\mathcal{A}$ is **perfect localizing** if S is exact.

$$\mathcal{C} \underset{S}{\overset{T}{\rightleftarrows}} \mathcal{C}/\mathcal{A}$$

Proposition

- *T is an exact functor.*
- *S is a fully faithful and left exact functor.*

Localization in coalgebras

$\mathcal{A}$ dense subcategory of an abelian category $\mathcal{C}$:

- There is a **quotient functor** $T : \mathcal{C} \to \mathcal{C}/\mathcal{A}$.
- $\mathcal{A}$ is **localizing** if T has a right adjoint functor $S : \mathcal{C}/\mathcal{A} \to \mathcal{C}$ (**section functor**).
- $\mathcal{A}$ is **perfect localizing** if S is exact.

$$\mathcal{C} \underset{S}{\overset{T}{\rightleftarrows}} \mathcal{C}/\mathcal{A}$$

Proposition

- *T is an exact functor.*
- *S is a fully faithful and left exact functor.*

C be a coalgebra and $\mathcal{M}^C$ the right C-comodules.

Theorem

There are one-to-one correspondences between:

- ***Localizing subcategories*** *of $\mathcal{M}^C$.*
- *Classes of equivalence of **injective** C-comodules.*
- ***Coidempotent subcoalgebras*** *of C $(A \wedge A = A)$.*
- *Sets of **indecomposable injective** C-comodules.*
- *Sets of **simple** C-comodules.*
- *Classes of equivalence of **idempotents** in C^*.*

Corollary

$\mathcal{M}^C/\mathcal{T}_e \simeq \mathcal{M}^{eCe}$

C be a coalgebra and $\mathcal{M}^C$ the right C-comodules.

Theorem

There are one-to-one correspondences between:

- ***Localizing subcategories*** *of* $\mathcal{M}^C$.
- *Classes of equivalence of* ***injective*** C*-comodules.*
- ***Coidempotent subcoalgebras*** *of* C $(A \wedge A = A)$.
- *Sets of* ***indecomposable injective*** C*-comodules.*
- *Sets of* ***simple*** C*-comodules.*
- *Classes of equivalence of* ***idempotents*** *in* C^*.

Corollary

$\mathcal{M}^C / \mathcal{T}_e \simeq \mathcal{M}^{eCe}$

Example: localization of path coalgebras

$Q = (Q_0, Q_1)$ quiver
e idempotent in $(KQ)^*$
$X \subseteq Q_0$ vertices associated to e

$$X \longleftrightarrow e(p) = \begin{cases} 1 & \text{if } p \in X \\ 0 & \text{otherwise} \end{cases}$$

Let p be a path in Q

$$\underset{x_1}{\circ} \longrightarrow \underset{x_2}{\bullet} \longrightarrow \bullet - \cdots - \bullet \longrightarrow \underset{x_{n-1}}{\bullet} \longrightarrow \underset{x_n}{\circ}$$

- p is a **cell** relative to X if $\begin{cases} x_1, x_n \in X, \\ x_2, x_3, \ldots, x_{n-1} \notin X \end{cases}$

Example: localization of path coalgebras

$Q = (Q_0, Q_1)$ quiver
e idempotent in $(KQ)^*$
$X \subseteq Q_0$ vertices associated to e

$$X \longleftrightarrow e(p) = \begin{cases} 1 & \text{if } p \in X \\ 0 & \text{otherwise} \end{cases}$$

Let p be a path in Q

$$\underset{x_1}{\circ} \longrightarrow \underset{x_2}{\bullet} \longrightarrow \bullet - \cdots - \bullet \longrightarrow \underset{x_{n-1}}{\bullet} \longrightarrow \underset{x_n}{\circ}$$

- p is a **cell** relative to X if $\begin{cases} x_1, x_n \in X, \\ x_2, x_3, \ldots, x_{n-1} \notin X \end{cases}$

Example: localization of path coalgebras

$Q = (Q_0, Q_1)$ quiver
e idempotent in $(KQ)^*$
$X \subseteq Q_0$ vertices associated to e

$$X \longleftrightarrow e(p) = \begin{cases} 1 & \text{if } p \in X \\ 0 & \text{otherwise} \end{cases}$$

Let p be a path in Q

$$\begin{array}{ccccccccccc} \circ & \longrightarrow & \bullet & \longrightarrow & \bullet & - \cdots - & \bullet & \longrightarrow & \bullet & \longrightarrow & \circ \\ x_1 & & x_2 & & & & & & x_{n-1} & & x_n \end{array}$$

- p is a **cell** relative to X if $\begin{cases} x_1, x_n \in X, \\ x_2, x_3, \ldots, x_{n-1} \notin X \end{cases}$

Example: localization of path coalgebras

Theorem

$e(KQ)e \cong KQ^e$, where $Q^e = (X, \mathcal{C}ell_X^Q)$.

Example

$$e(\circ) = 1 \text{ and } e(\bullet) = 0$$

$$\mathbb{A}_3 : \circ \longrightarrow \bullet \longrightarrow \circ$$

$$\rightsquigarrow$$

$$(\mathbb{A}_3)^e : \circ \qquad\qquad \circ$$

Example: localization of path coalgebras

Theorem

$e(KQ)e \cong KQ^e$, where $Q^e = (X, \mathcal{C}ell_X^Q)$.

Example

$$e(\circ) = 1 \text{ and } e(\bullet) = 0$$

Example: localization of path coalgebras

Theorem

$e(KQ)e \cong KQ^e$, where $Q^e = (X, \mathcal{C}ell_X^Q)$.

Example

$$e(\circ) = 1 \text{ and } e(\bullet) = 0$$

$$\mathbb{A}_3 : \circ \longrightarrow \bullet \longrightarrow \circ$$

$$\rightsquigarrow$$

$$(\mathbb{A}_3)^e : \circ \qquad\qquad \circ$$

Example: localization of path coalgebras

Theorem

$e(KQ)e \cong KQ^e$, where $Q^e = (X, \mathcal{C}ell_X^Q)$.

Example

$$e(\circ) = 1 \text{ and } e(\bullet) = 0$$

$$\mathbb{A}_3 : \ \circ \longrightarrow \bullet \longrightarrow \circ$$

$$\rightsquigarrow$$

$$(\mathbb{A}_3)^e : \ \circ \qquad\qquad \circ$$

Example: localization of path coalgebras

Theorem

$e(KQ)e \cong KQ^e$, where $Q^e = (X, \mathcal{C}ell_X^Q)$.

Example

$$e(\circ) = 1 \text{ and } e(\bullet) = 0$$

$$\mathbb{A}_3 : \ \circ \longrightarrow \bullet \longrightarrow \circ$$

$$\rightsquigarrow$$

$$(\mathbb{A}_3)^e : \ \circ \longrightarrow \circ$$

Example: localization of path coalgebras

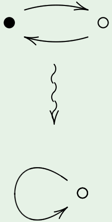
Example

$$e(\circ) = 1 \text{ and } e(\bullet) = 0$$

Example: localization of path coalgebras

Example

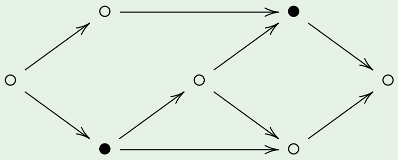
$$e(\circ) = 1 \text{ and } e(\bullet) = 0$$



Example: localization of path coalgebras

Example

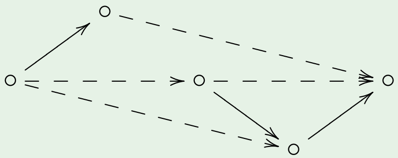
$$e(\circ) = 1 \text{ and } e(\bullet) = 0$$



Example: localization of path coalgebras

Example

$$e(\circ) = 1 \text{ and } e(\bullet) = 0$$



Proposition

If M is a uniserial C-comodule, then $T(M) = eM$ is a uniserial eCe-comodule

Corollary

- *If C is serial, then eCe is serial.*
- *If any socle-finite eCe is serial then C is serial.*

Example

KQ is not serial

Some consequences

Proposition

If M is a uniserial C-comodule, then $T(M) = eM$ is a uniserial eCe-comodule

Corollary

- *If C is serial, then eCe is serial.*
- *If any socle-finite eCe is serial then C is serial.*

Example

KQ is not serial

Some consequences

Proposition

If M is a uniserial C-comodule, then $T(M) = eM$ is a uniserial eCe-comodule

Corollary

- *If C is serial, then eCe is serial.*
- *If any socle-finite eCe is serial then C is serial.*

Example

KQ is not serial

Some consequences

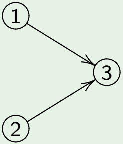
Proposition

If M is a uniserial C-comodule, then $T(M) = eM$ is a uniserial eCe-comodule

Corollary

- *If C is serial, then eCe is serial.*
- *If any socle-finite eCe is serial then C is serial.*

Example



KQ is not serial

Eisenbud-Griffith Theorem for coalgebras

Theorem (Eisenbud-Griffith Theorem)

Every proper quotient of a hereditary noetherian prime ring is serial

"Coalgebraic" notions:

- Subcoalgebra
- hereditary (global dimension 0 or 1)
- Strictly quasi-finite (quotients are quasi-finite comodules)
- prime ($A \wedge B = C$, then $A = C$ or $B = C$).

Theorem (Eisenbud-Griffith Theorem)

Every proper ***quotient*** *of a hereditary noetherian prime ring is serial*

"Coalgebraic" notions:

- **Subcoalgebra**
- hereditary (global dimension 0 or 1)
- Strictly quasi-finite (quotients are quasi-finite comodules)
- prime ($A \wedge B = C$, then $A = C$ or $B = C$).

Eisenbud-Griffith Theorem for coalgebras

Theorem (Eisenbud-Griffith Theorem)

*Every proper quotient of a **hereditary** noetherian prime ring is serial*

"Coalgebraic" notions:

- Subcoalgebra
- **Hereditary** (global dimension 0 or 1)
- Strictly quasi-finite (quotients are quasi-finite comodules)
- prime ($A \wedge B = C$, then $A = C$ or $B = C$).

Eisenbud-Griffith Theorem for coalgebras

Theorem (Eisenbud-Griffith Theorem)

Every proper quotient of a hereditary ***noetherian*** *prime ring is serial*

"Coalgebraic" notions:

- Subcoalgebra
- hereditary (global dimension 0 or 1)
- **Strictly quasi-finite** (quotients are quasi-finite comodules)
- prime ($A \wedge B = C$, then $A = C$ or $B = C$).

Eisenbud-Griffith Theorem for coalgebras

Theorem (Eisenbud-Griffith Theorem)

Every proper quotient of a hereditary noetherian ***prime*** *ring is serial*

"Coalgebraic" notions:

- Subcoalgebra
- hereditary (global dimension 0 or 1)
- Strictly quasi-finite (quotients are quasi-finite comodules)
- **prime** ($A \wedge B = C$, then $A = C$ or $B = C$).

Eisenbud-Griffith Theorem for coalgebras

Theorem

Every subcoalgebra of a prime, hereditary and strictly quasi-finite coalgebra is serial.

Eisenbud-Griffith Theorem for coalgebras

STEP 1: Reduction to socle-finite coalgebras

eCe socle-finite "localized" coalgebra

$\Downarrow$

eCe is $\begin{cases} \textit{hereditary} \\ \text{strictly quasi-finite} \\ \textit{prime} \end{cases}$

$\Downarrow$ E-G theorem

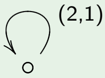
eCe is serial

$\Downarrow$

C is serial

STEP 2: The colocal case

Example

If 

then C is **NOT** strictly quasi-finite

Therefore,

Lemma

The quiver of a colocal "localized" coalgebra is:

- *a single point, or*
-

STEP 2: The colocal case

Example

If  (2,1) then C is **NOT** strictly quasi-finite

Therefore,

Lemma

The quiver of a colocal "localized" coalgebra is:

- *a single point, or*
-  *(1,1)*

Eisenbud-Griffith Theorem for coalgebras

STEP 3: Deduction of the quiver

Theorem

- *If C is prime, then each vertex is in a cycle.*
- *If C is hereditary and*

$$x \xrightarrow{(d_1,d_2)} \bullet \xrightarrow{(c_1,c_2)} y$$

then

$$x \xrightarrow{(c_1 d_1, c_2 d_2)} y$$

Corollary

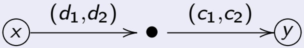
The quiver of C is

Eisenbud-Griffith Theorem for coalgebras

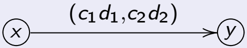
STEP 3: Deduction of the quiver

Theorem

- *If C is prime, then each vertex is in a cycle.*
- *If C is hereditary and*



then



Corollary

The quiver of C is