Ejemplo de la DVS de una matriz

$$A_{3\times 4} = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{pmatrix} \qquad r(A) = 2 \Longrightarrow A_{3\times 4} = V_{3\times 2}\mathbb{D}_{\alpha 2\times 2}U'_{2\times 4} \qquad \vec{v}_1, \vec{v}_2 \in \mathbb{R}^3 \qquad \vec{u}_1, \vec{u}_2 \in \mathbb{R}^4$$

$$A = \left(\vec{v}_1, \ \vec{v}_2 \right) \mathbb{D}_\alpha \begin{pmatrix} \vec{u}'_1 \\ \vec{u}'_2 \end{pmatrix} = \begin{pmatrix} v_{11} & v_{21} \\ v_{12} & v_{22} \\ v_{13} & v_{23} \end{pmatrix} \begin{pmatrix} \alpha_1 & \\ & \alpha_2 \end{pmatrix} \begin{pmatrix} u_{11} & u_{12} & u_{13} & u_{14} \\ u_{21} & u_{22} & u_{23} & u_{24} \end{pmatrix} =$$

$$= \alpha_1 \begin{pmatrix} v_{11} \\ v_{12} \\ v_{13} \end{pmatrix} (u_{11}, u_{12}, u_{13}, u_{14}) + \alpha_2 \begin{pmatrix} v_{21} \\ v_{22} \\ v_{23} \end{pmatrix} (u_{21}, u_{22}, u_{23}, u_{24}) = \alpha_1 \vec{v}_1 \vec{u}'_1 + \alpha_2 \vec{v}_2 \vec{u}'_2 = \sum_{k=1}^{2} \alpha_k \vec{v}_k \vec{u}'_k$$

$$A_{3\times 4} = \begin{pmatrix} 1 & 2 & 3 & 5 \\ 4 & 1 & 2 & 3 \\ 5 & 3 & 5 & 8 \end{pmatrix} \qquad r(A) = 2\ ; \ (1^{\underline{a}} + 2^{\underline{a}} = 3^{\underline{a}})$$

$$A = \begin{pmatrix} -0.4389020 & 0.68849958 \\ -0.3768071 & -0.72435010 \\ \underbrace{-0.8157091}_{\vec{v}_1} & \underbrace{-0.03585052}_{\vec{v}_2} \end{pmatrix} \begin{pmatrix} \underbrace{13.59568}_{\alpha_1} & \\ & \underbrace{2.67534}_{\alpha_2} \end{pmatrix} \begin{pmatrix} -0.4431316 & -0.8926543 \\ -0.2722731 & 0.2037489 \\ -0.4522661 & 0.1635478 \\ \underbrace{-0.7245392}_{\vec{u}_1} & \underbrace{0.3672967}_{\vec{u}_2} \end{pmatrix}'$$

DVS de una matriz cuadrada real y simétrica

$B_{4\times 4}$

$$B=\begin{pmatrix}1&2&3&4\\2&3&4&5\\3&4&4&5\\4&5&5&6\end{pmatrix}\qquad\begin{cases}r(B)=3\\(2^{\underline{a}}+3^{\underline{a}}-1^{\underline{a}}=4^{\underline{a}})\end{cases}\qquad B=\begin{pmatrix}0.3392814&0.697213&0.3857230\\0.4630156&0.4517884&-0.5757637\\0.5138280&-0.2512294&0.6502804\\\underbrace{0.6375622}_{\vec{v}_1}&\underbrace{-0.4966539}_{\vec{v}_2}&\underbrace{-0.3112063}_{\vec{v}_3}\end{pmatrix}\cdot$$

$$\cdot\begin{pmatrix}\underbrace{15.78939}_{\lambda_1}&&\\&\underbrace{-1.634384}_{\lambda_2}&\\&&\underbrace{-0.1550031}_{\lambda_3}\end{pmatrix}\cdot\begin{pmatrix}0.3392814&0.697213&0.3857230\\0.4630156&0.4517884&-0.5757637\\0.5138280&-0.2512294&0.6502804\\\underbrace{0.6375622}_{\vec{v}_1}&\underbrace{-0.4966539}_{\vec{v}_2}&\underbrace{-0.3112063}_{\vec{v}_3}\end{pmatrix}'$$

Existencia de la DVS de una matriz A

$$A = \begin{pmatrix} 1 & 2 & 3 & 5 \\ 4 & 1 & 2 & 3 \\ 5 & 3 & 5 & 8 \end{pmatrix} == \begin{pmatrix} -0.4389020 & 0.68849958 \\ -0.3768071 & -0.72435010 \\ -0.8157091 & -0.03585052 \end{pmatrix} \begin{pmatrix} 13.59568 & \\ & 2.67534 \end{pmatrix} \begin{pmatrix} -0.4431316 & -0.8926543 \\ -0.2722731 & 0.2037489 \\ -0.4522661 & 0.1635478 \\ -0.7245392 & 0.3672967 \end{pmatrix}'$$

$$AA' = \begin{pmatrix} 39 & 27 & 66 \\ 27 & 30 & 57 \\ 66 & 57 & 123 \end{pmatrix} =$$

$$= \begin{pmatrix} -0.4389020 & 0.68849958 \\ -0.3768071 & -0.72435010 \\ -0.8157091 & -0.03585052 \end{pmatrix} \begin{pmatrix} 184.8426 & \\ & 7.157443 \end{pmatrix} \begin{pmatrix} -0.4389020 & -0.3768071 & -0.8157091 \\ 0.68849958 & -0.72435010 & -0.03585052 \end{pmatrix}$$

$$A'A = \begin{pmatrix} 42 & 21 & 36 & 57 \\ 21 & 14 & 23 & 37 \\ 36 & 23 & 38 & 61 \\ 57 & 37 & 61 & 98 \end{pmatrix} =$$

$$= \begin{pmatrix} -0.4431316 & -0.8926543 \\ -0.2722731 & 0.2037489 \\ -0.4522661 & 0.1635478 \\ -0.7245392 & 0.3672967 \end{pmatrix} \begin{pmatrix} 184.8426 & \\ & 7.157443 \end{pmatrix} \begin{pmatrix} -0.4431316 & -0.8926543 \\ -0.2722731 & 0.2037489 \\ -0.4522661 & 0.1635478 \\ -0.7245392 & 0.3672967 \end{pmatrix}'$$