

Método de WARD

Individuo	X_1	X_2
A	10	5
B	20	20
C	30	10
D	30	15
E	5	10

Nivel 1

Combinaciones posibles: $\binom{5}{2} = 10$

Partición	Centroides	E_k	E	ΔE
$(A, B), C, D, E$	$C_{AB} = (15, 12.5)$	$E_{AB} = 162.5$ $E_C = E_D = E_E = 0$	162.5	162.5
$(A, C), B, D, E$	$C_{AC} = (20, 7.5)$	$E_{AC} = 212.5$ $E_B = E_D = E_E = 0$	212.5	212.5
$(A, D), B, C, E$	$C_{AD} = (20, 10)$	$E_{AD} = 250$ $E_B = E_C = E_E = 0$	250	250
$(A, E), B, C, D$	$C_{AE} = (7.5, 7.5)$	$E_{AE} = 25$ $E_B = E_C = E_D = 0$	25	25
$(B, C), A, D, E$	$C_{BC} = (25, 15)$	$E_{BC} = 100$ $E_A = E_D = E_E = 0$	100	100
$(B, D), A, C, E$	$C_{BD} = (25, 17.5)$	$E_{BD} = 62.5$ $E_A = E_C = E_E = 0$	62.5	62.5
$(B, E), A, C, D$	$C_{BE} = (12.5, 15)$	$E_{BE} = 162.5$ $E_A = E_C = E_D = 0$	162.5	162.5
$(C, D), A, B, E$	$C_{CD} = (30, 12.5)$	$E_{CD} = 12.5$ $E_A = E_B = E_E = 0$	12.5	12.5
$(C, E), A, B, D$	$C_{CE} = (17.5; 10)$	$E_{CE} = 312.5$ $E_A = E_B = E_D = 0$	312.5	312.5
$(D, E), A, B, C$	$C_{DE} = (17.5; 12.5)$	$E_{DE} = 325$ $E_A = E_B = E_C = 0$	325	325

Cluster $(C, D), A, B, E$

$$E_{AB} = (10 - 15)^2 + (5 - 12.5)^2 + (20 - 15)^2 + (20 - 12.5)^2 = 162.5$$

$$E_{AB} = 10^2 + 5^2 + 20^2 + 20^2 - 2(15^2 + 12.5^2) = 162.5$$

Nivel 2

Combinaciones posibles: $\binom{4}{2} = 6$

Partición	Centroides	E_k	E	ΔE
$((C, D), A), B, E$	$C_{CDA} = (23.33, 10)$	$E_{CDA} = 316.66$ $E_B = E_E = 0$	316.66	304.16
$((C, D), B), A, E$	$C_{CDB} = (26.66, 15)$	$E_{CDB} = 116.66$ $E_A = E_E = 0$	116.66	104.16
$((C, D), E), A, B$	$C_{CDE} = (21.66, 11.66)$	$E_{CDE} = 433.33$ $E_A = E_B = 0$	433.33	420.83
$(A, B), (C, D), E$	$C_{AB} = (15, 12.5)$ $C_{CD} = (30, 12.5)$	$E_{AB} = 162.5$ $E_{CD} = 12.5$ $E_E = 0$	175	162.5
$(A, E), (C, D), B$	$C_{AE} = (7.5, 7.5)$ $C_{CD} = (30, 12.5)$	$E_{AE} = 25$ $E_{CD} = 12.5$ $E_B = 0$	37.5	25
$(B, E), (C, D), A$	$C_{BE} = (12.5, 15)$ $C_{CD} = (30, 12.5)$	$E_{BE} = 162.5$ $E_{CD} = 12.5$ $E_A = 0$	175	162.5

Cluster $(A, E), (C, D), B$

Cálculos para la partición $((C, D), A), B, E$:

$$m_{(C,D),A} = \frac{2(m^{CD}) + m^A}{3} = \frac{2(30, 12.5) + (10, 5)}{3} = (23.33, 10)$$

$$E_{(C,D),A} = \sum_{i=1}^3 \sum_{l=1}^2 (x_{il}^{(C,D),A} - m_l^{(C,D),A})^2 =$$

$$= (30 - 23.33)^2 + (10 - 10)^2 + (30 - 23.33)^2 + (15 - 10)^2 + (10 - 23.33)^2 + (5 - 10)^2 = 316.66$$

$$E_{(C,D),A} = \sum_{i=1}^3 \sum_{l=1}^2 (x_{il}^{(C,D),A})^2 - 3 \sum_{l=1}^2 (m_l^{(C,D),A})^2 =$$

$$= 30^2 + 10^2 + 30^2 + 15^2 + 10^2 + 5^2 - 3(23.33^2 + 10^2) = 316.66$$

$$\Delta E_{(C,D),A} = E_{(C,D),A} - E_{CD} - E_A = 316.66 - 12.5 - 0 = 304.16$$

$$\Delta E_{(C,D),A} = \frac{n_{CD}n_A}{n_{CD} + n_A} \sum_{l=1}^2 (m_l^{CD} - m_l^A)^2 = \frac{1 \cdot 2}{3} [(30 - 10)^2 + (12.5 - 5)^2] = 304.16$$

$$\Delta E_{(C,D),A} = \frac{1}{n_{CD} + n_A} [(n_C + n_A)\Delta E_{CA} + (n_D + n_A)\Delta E_{DA} - n_A\Delta E_{CD}] =$$

$$= \frac{1}{2 + 1} (2 \cdot 212.5 + 2 \cdot 250 - 12.5) = 304.16$$

Nivel 3

Combinaciones posibles: $\binom{3}{2} = 3$

Partición	Centroides	E_k	E	ΔE
$((A, E), (C, D)), B$	$C_{AECD} = (18.75, 10)$	$E_{AECD} = 568.75$ $E_B = 0$	568.75	531.25
$((A, E), B), (C, D)$	$C_{AEB} = (11.66, 11.66)$ $C_{CD} = (30, 12.5)$	$E_{AEB} = 233.33$ $E_{CD} = 12.5$	245.8	208.3
$((C, D), B), (A, E)$	$C_{CDB} = (26.66, 15)$ $C_{AE} = (7.5, 7.5)$	$E_{CDB} = 116.66$ $E_{AE} = 25$	141.66	104.16

Cluster $((C, D), B), (A, E)$

Nivel 4

Partición	Centroide	E	ΔE
$((((C, D), B), (A, E)))$	$C_{CDBAE} = (19, 12)$	650	508.34

