1-Práctica de Reconstrucción de una matriz con R

Pasos a seguir en la realización de la práctica:

- Los datos del fichero pesos.txt, corresponden a los pesos de 20 terneros en tres momentos, al nacer, a los 30 días y a los 90 días. Vamos a tratar de reconstruir esta tabla de datos, aplicando la teoría de la reconstrucción del Análisis Factorial General.

```
#matriz de datos por columnas
J1 <- read.table("pesos.txt")
J1

##    V1  V2  V3
## 1  58 116 178
## 2  45  79 125
## 3  55  98 162
## 4  51  92 141
## 5  51  87 136
## 6  55  99 156
## 7  46  85 137
## 8  45  79 126
## 9  45  80 135
## 10 53  98 158
## 11 45  74 103
## 12 46  85 132
## 13 53  93 143
## 14 44  90 153
## 15 52  94 148
## 16 46  89 148
## 17 53  92 143
## 18 50  96 154
## 19 52  91 152
## 20 50  76 127
```

- Convertimos la tabla J1 en una matriz.

```
J1.m <- as.matrix(J1)
J1.m

##       V1  V2  V3
##  [1,] 58 116 178
##  [2,] 45  79 125
##  [3,] 55  98 162
##  [4,] 51  92 141
##  [5,] 51  87 136
##  [6,] 55  99 156
##  [7,] 46  85 137
##  [8,] 45  79 126
##  [9,] 45  80 135
## [10,] 53  98 158
## [11,] 45  74 103
## [12,] 46  85 132
## [13,] 53  93 143
## [14,] 44  90 153
## [15,] 52  94 148
## [16,] 46  89 148
## [17,] 53  92 143
## [18,] 50  96 154
## [19,] 52  91 152
## [20,] 50  76 127
```

- Calculamos en J2, el producto $X'X_{3\times3}$.

```
J2 <- t(J1.m)%*%J1.m
J2

##        V1     V2     V3
## V1  49835  89825 143042
## V2  89825 162549 258949
## V3 143042 258949 413197
```

- Calculamos los valores y vectores propios de $X'X$.

```
eigen(J2)

## $values
## [1] 625144.4641    334.8611    101.6748
##
## $vectors
##            [,1]       [,2]       [,3]
## [1,] -0.2817030  0.7525292  0.5952673
## [2,] -0.5097409  0.4082291 -0.7573066
## [3,] -0.8129008 -0.5167676  0.2685955
```

- Calculamos en J3, el producto $XX'_{20\times20}$.

```
J3 <- J1.m%*%t(J1.m)
J3

##        [,1]  [,2]  [,3]  [,4]  [,5]  [,6]  [,7]  [,8]  [,9] [,10] [,11]
##  [1,] 48504 34024 43394 38728 37258 42442 36914 34202 35920 42566 29528
##  [2,] 34024 23891 30467 27188 26168 29796 25910 24016 25220 29877 20746
##  [3,] 43394 30467 38873 34663 33363 37999 33054 30629 32185 38115 26413
##  [4,] 38728 27188 34663 30946 29781 33909 29483 27329 28690 33997 23626
##  [5,] 37258 26168 33363 29781 28666 32634 28373 26304 27615 32717 22741
##  [6,] 42442 29796 37999 33909 32634 37162 32317 29952 31455 37265 25869
##  [7,] 36914 25910 33054 29483 28373 32317 28110 26047 27365 32414 22471
##  [8,] 34202 24016 30629 27329 26304 29952 26047 24142 25355 30035 20849
##  [9,] 35920 25220 32185 28690 27615 31455 27365 25355 26650 31555 21850
## [10,] 42566 29877 38115 33997 32717 37265 32414 30035 31555 37377 25911
## [11,] 29528 20746 26413 23626 22741 25869 22471 20849 21850 25911 18110
## [12,] 36024 25285 32244 28778 27693 31537 27425 25417 26690 31624 21956
## [13,] 39316 27607 35195 31422 30242 34430 29934 27750 29130 34517 23996
## [14,] 40226 28215 36026 32097 30882 35198 30635 28368 29835 35326 24399
## [15,] 40264 28266 36048 32168 30958 35254 30658 28414 29840 35352 24540
## [16,] 39336 27601 35228 31402 30217 34429 29957 27749 29170 34544 23900
## [17,] 39200 27528 35097 31330 30155 34331 29849 27671 29050 34419 23922
## [18,] 41448 29084 37106 33096 31846 36278 31558 29238 30720 36390 25216
## [19,] 40628 28529 36402 32456 31241 35581 30951 28681 30140 35690 24730
## [20,] 34322 24129 30772 27449 26434 30086 26159 24256 25475 30164 20955
##       [,12] [,13] [,14] [,15] [,16] [,17] [,18] [,19] [,20]
##  [1,] 36024 39316 40226 40264 39336 39200 41448 40628 34322
##  [2,] 25285 27607 28215 28266 27601 27528 29084 28529 24129
##  [3,] 32244 35195 36026 36048 35228 35097 37106 36402 30772
##  [4,] 28778 31422 32097 32168 31402 31330 33096 32456 27449
##  [5,] 27693 30242 30882 30958 30217 30155 31846 31241 26434
##  [6,] 31537 34430 35198 35254 34429 34331 36278 35581 30086
##  [7,] 27425 29934 30635 30658 29957 29849 31558 30951 26159
```

```
##  [8,] 25417 27750 28368 28414 27749 27671 29238 28681 24256
##  [9,] 26690 29130 29835 29840 29170 29050 30720 30140 25475
## [10,] 31624 34517 35326 35352 34544 34419 36390 35690 30164
## [11,] 21956 23996 24399 24540 23900 23922 25216 24730 20955
## [12,] 26765 29219 29870 29918 29217 29134 30788 30191 25524
## [13,] 29219 31907 32581 32662 31879 31814 33600 32955 27879
## [14,] 29870 32581 33445 33392 32678 32491 34402 33734 28471
## [15,] 29918 32662 33392 33444 32662 32568 34416 33754 28540
## [16,] 29217 31879 32678 32662 31941 31790 33636 32987 27860
## [17,] 29134 31814 32491 32568 31790 31722 33504 32864 27803
## [18,] 30788 33600 34402 34416 33636 33504 35432 34744 29354
## [19,] 30191 32955 33734 33754 32987 32864 34744 34089 28820
## [20,] 25524 27879 28471 28540 27860 27803 29354 28820 24405
```

- Calculamos los valores y vectores propios de XX'.

```
eigen(J3)

## $values
##  [1]  6.251445e+05  3.348611e+02  1.016748e+02  1.856644e-11  3.964113e-12
##  [6]  2.907398e-12  2.549756e-12  2.242741e-12  1.281282e-12  1.231247e-12
## [11]  1.157995e-12  3.943690e-13  3.292269e-13  3.126965e-13  1.624245e-13
## [16] -5.823384e-14 -1.220097e-12 -2.009409e-12 -2.371529e-12 -1.823261e-10
##
## $vectors
##             [,1]        [,2]        [,3]         [,4]        [,5]
##  [1,] -0.2784570  0.05373856  0.54664700  0.000000000  0.00000000
##  [2,] -0.1954805 -0.08295200 -0.05298312  0.272980867  0.33873187
##  [3,] -0.2493335  0.12682526 -0.20192926  0.077390121  0.15302646
##  [4,] -0.2224494 -0.16786664  0.14296813 -0.332942952 -0.26629337
##  [5,] -0.2140853 -0.19752326 -0.09936652 -0.451426606 -0.08175424
##  [6,] -0.2438094 -0.06492262  0.03299958  0.267932480 -0.16530223
##  [7,] -0.2120425  0.08095168  0.01895953  0.204526609 -0.02886233
##  [8,] -0.1965086 -0.05471211 -0.07962053  0.350263424 -0.12880710
##  [9,] -0.2064065  0.17713830 -0.24425293 -0.026118597 -0.21183178
## [10,] -0.2445084  0.09611288  0.02268926 -0.085562812  0.62395345
## [11,] -0.1696382 -0.59268669  0.15751831 -0.032452959  0.03851869
## [12,] -0.2069019 -0.06024775  0.15214661  0.232724594 -0.37954900
## [13,] -0.2258629 -0.21594260  0.04672878  0.020140096 -0.06990121
## [14,] -0.2310035  0.50349423  0.08635146 -0.321489705 -0.05812765
## [15,] -0.2312920 -0.05592815  0.04768048  0.202079027  0.25054262
## [16,] -0.2259308  0.30235618  0.02636533 -0.001806629  0.03174313
## [17,] -0.2252182 -0.19363404 -0.02837556 -0.376522547  0.19349824
## [18,] -0.2380376  0.15114122  0.15613353  0.105926831 -0.07984712
## [19,] -0.2334704  0.12395708 -0.28418222 -0.081998402 -0.20823610
## [20,] -0.1973841 -0.16516447 -0.62674313  0.031949859 -0.02330668
##              [,6]        [,7]        [,8]         [,9]        [,10]
##  [1,]  0.00000000  0.00000000  0.00000000  0.000000000  0.000000000
##  [2,] -0.11201378  0.02870829 -0.02974061  0.007763384 -0.000722014
##  [3,] -0.19634439 -0.16744062  0.19054111  0.184390302 -0.125895458
##  [4,] -0.16664805 -0.07464775  0.02200087 -0.263346211  0.047490058
##  [5,]  0.14269535 -0.35385394  0.24360412 -0.012668528 -0.008558043
##  [6,] -0.29247160  0.32378095  0.36566068  0.023540136  0.217595690
##  [7,] -0.42035863 -0.33583583 -0.09965907  0.036495817  0.020224984
##  [8,] -0.06536185 -0.14567116 -0.59452737 -0.084411990 -0.081246500
##  [9,]  0.07701671 -0.01863738  0.22753826 -0.276025811 -0.075032566
## [10,]  0.14198484 -0.26979352  0.06894074  0.094202348 -0.126869892
## [11,]  0.10189634  0.33283657 -0.10768694  0.013292803 -0.530568777
```

```
## [12,]  0.07880304 -0.31960950  0.21593933  0.288898312 -0.308681127
## [13,]  0.22869767 -0.24424784 -0.24097153 -0.248205454  0.523280226
## [14,] -0.22143461  0.21404962 -0.13275845 -0.119685023 -0.213303925
## [15,]  0.01178196  0.23488931  0.26603179 -0.503548079  0.098031888
## [16,]  0.42422481  0.06546336 -0.21639338 -0.181353392 -0.168134441
## [17,] -0.29261650  0.16360280 -0.21167527  0.337110076  0.270729925
## [18,]  0.44967021  0.21422826  0.08801034  0.460015643  0.312211510
## [19,] -0.01332983  0.26858132 -0.20907403  0.175330959 -0.004146976
## [20,]  0.14874206  0.07563158  0.03964948  0.018297119 -0.031017742
##              [,11]       [,12]        [,13]        [,14]       [,15]
##  [1,]  0.000000000  0.00000000  0.000000000  0.000000000  0.00000000
##  [2,]  0.008033111 -0.01954981  0.001632634 -0.003626155  0.01476251
##  [3,]  0.107480716  0.06578271 -0.300171063 -0.212866538  0.05193585
##  [4,]  0.371705852 -0.30183380 -0.306874845 -0.247753293 -0.30166321
##  [5,] -0.243807912  0.06677974  0.372609120 -0.097941322  0.04893417
##  [6,] -0.284985605 -0.05219831 -0.314760542  0.123935674  0.15176116
##  [7,]  0.296867885  0.05088696  0.369825097  0.532320119 -0.19786470
##  [8,] -0.222386323  0.17318464  0.005754525 -0.489556734 -0.14862756
##  [9,] -0.263872946  0.55429679 -0.063752272  0.120484296 -0.17711227
## [10,]  0.001915082 -0.03713822 -0.163141033 -0.064535623  0.17893354
## [11,]  0.124757182  0.26059477 -0.002492566  0.197509672  0.09703092
## [12,] -0.145322478 -0.25469026  0.024266555 -0.047088783  0.10055008
## [13,]  0.086422057  0.12694300 -0.190455639  0.198994438  0.46428919
## [14,]  0.192536918  0.22156849 -0.002104030 -0.056127601  0.26238507
## [15,]  0.002589075 -0.17787016  0.462298818 -0.242738905 -0.08068914
## [16,] -0.264368586 -0.37410511 -0.177056395  0.383801649 -0.28635035
## [17,] -0.375004264 -0.01448108  0.004949272  0.071899034 -0.30060647
## [18,]  0.309246658  0.25162980  0.137213186 -0.150824665 -0.24448171
## [19,] -0.027611017 -0.33214862  0.312518638 -0.039340657  0.43924228
## [20,]  0.337929707 -0.11121937 -0.108930271  0.083432517 -0.14324521
##             [,16]        [,17]        [,18]         [,19]        [,20]
##  [1,]  0.77214439  0.000000000  0.000000000  0.0000000000  0.156665071
##  [2,]  0.14565740  0.019016919  0.006580838 -0.0172362046 -0.852013005
##  [3,]  0.01418913 -0.201943601  0.370031180  0.5873900705  0.147983871
##  [4,] -0.13952424  0.040517812  0.215200813 -0.2219397290 -0.148992563
##  [5,]  0.03406074  0.430904994 -0.039374880  0.2619296695 -0.133918377
##  [6,] -0.12811265  0.453256078 -0.092402292 -0.0359328129  0.105197356
##  [7,] -0.12020925  0.125731534  0.090127979 -0.0333563220  0.121659329
##  [8,] -0.03751959  0.233515959 -0.070795604 -0.0205547704  0.132229925
##  [9,]  0.09978815 -0.254907884  0.237559524 -0.3438372649 -0.067182944
## [10,] -0.15704235  0.148437771  0.001309974 -0.4986188325  0.227276593
## [11,] -0.14462531  0.010166096  0.126325274  0.0710863620  0.064966012
## [12,] -0.16027266 -0.391756620 -0.315592505 -0.1142217495 -0.088038188
## [13,] -0.09872408 -0.216186807 -0.052354006  0.1231480467 -0.003852713
## [14,] -0.15551136  0.004745141 -0.444500687  0.1128448229 -0.118138354
## [15,] -0.15039446 -0.257428527 -0.077827107  0.0949987860  0.182953764
## [16,] -0.11359037  0.094634963  0.073972180  0.2537069879 -0.037433738
## [17,] -0.05885891 -0.349665229 -0.149658173 -0.0076794661  0.055219743
## [18,] -0.19558346  0.093751840  0.056011763  0.0002160233 -0.055766149
## [19,]  0.10617740 -0.029054459  0.436399189 -0.2158334027  0.010789421
## [20,]  0.35469913  0.035845807 -0.440086085 -0.0332558346  0.144518673
```

- Se comprueba que los autovalores de $X'X$ y XX' coinciden y comenzamos la reconstrucción de la matriz X a través de la expresión $X_1 = \lambda_1 v_1 u_1'$ es decir con el primer término de la sumatoria.

```
X_1 <- (sqrt(eigen(J3)$values[1]))*(eigen(J3)$vectors[,1])%*%(t(eigen(J2)$vectors[,1]))
X_1
```

```
##           [,1]      [,2]     [,3]
```

```
##  [1,] 62.02116 112.22713 178.9723
##  [2,] 43.53967  78.78492 125.6409
##  [3,] 55.53443 100.48940 160.2538
##  [4,] 49.54650  89.65425 142.9746
##  [5,] 47.68354  86.28323 137.5987
##  [6,] 54.30404  98.26302 156.7033
##  [7,] 47.22856  85.45995 136.2858
##  [8,] 43.76867  79.19929 126.3017
##  [9,] 45.97323  83.18844 132.6634
## [10,] 54.45973  98.54473 157.1526
## [11,] 37.78377  68.36963 109.0313
## [12,] 46.08358  83.38811 132.9818
## [13,] 50.30680  91.03002 145.1686
## [14,] 51.45177  93.10184 148.4726
## [15,] 51.51602  93.21810 148.6580
## [16,] 50.32191  91.05735 145.2122
## [17,] 50.16321  90.77018 144.7542
## [18,] 53.01848  95.93680 152.9936
## [19,] 52.00122  94.09607 150.0581
## [20,] 43.96366  79.55212 126.8644
```

- Calculamos la reconstrucción de la matriz X con 2 autovalores, es decir con dos sumandos $X_2 = \sqrt{\lambda_1} v_1 u_1' + \sqrt{\lambda_2} v_2 u_2'$

```
X_2 <- (sqrt(eigen(J3)$values[1]))*(eigen(J3)$vectors[,1])%*%(t(eigen(J2)$vectors[,1])) +
+(sqrt(eigen(J3)$values[2]))*(eigen(J3)$vectors[,2])%*%(t(eigen(J2)$vectors[,2]))
X_2
```

```
##           [,1]      [,2]     [,3]
##  [1,] 62.76118 112.62857 178.4642
##  [2,] 42.39737  78.16524 126.4254
##  [3,] 57.28090 101.43681 159.0545
##  [4,] 47.23486  88.40025 144.5621
##  [5,] 44.96351  84.80768 139.4666
##  [6,] 53.41002  97.77803 157.3172
##  [7,] 48.34332  86.06468 135.5203
##  [8,] 43.01525  78.79057 126.8191
##  [9,] 48.41255  84.51171 130.9883
## [10,] 55.78327  99.26272 156.2437
## [11,] 29.62206  63.94210 114.6360
## [12,] 45.25393  82.93804 133.5515
## [13,] 47.33312  89.41687 147.2107
## [14,] 58.38524  96.86308 143.7113
## [15,] 50.74585  92.80030 149.1869
## [16,] 54.48556  93.31603 142.3530
## [17,] 47.49673  89.32368 146.5853
## [18,] 55.09980  97.06586 151.5643
## [19,] 53.70820  95.02206 148.8859
## [20,] 41.68923  78.31830 128.4263
```

- Sorprendentemente la aproximación con 2 autovalores ha empeorado. Vamos a realizar la reconstrucción completa de la matriz con los tres autovalores, $X_3 = \sqrt{\lambda_1} v_1 u_1' + \sqrt{\lambda_2} v_2 u_2' + \sqrt{\lambda_3} v_3 u_3'$ que nos debería devolver la matriz de partida inicial ya que X es de rango 3.

```
X_3 <- (sqrt(eigen(J3)$values[1]))*(eigen(J3)$vectors[,1])%*%(t(eigen(J2)$vectors[,1])) +
+(sqrt(eigen(J3)$values[2]))*(eigen(J3)$vectors[,2])%*%(t(eigen(J2)$vectors[,2])) +
+(sqrt(eigen(J3)$values[3]))*(eigen(J3)$vectors[,3])%*%(t(eigen(J2)$vectors[,3]))
X_3
```

```
##            [,1]      [,2]     [,3]
##  [1,] 66.04233 108.45425 179.9447
##  [2,] 42.07934  78.56983 126.2819
##  [3,] 56.06886 102.97879 158.5076
##  [4,] 48.09300  87.30851 144.9493
##  [5,] 44.36708  85.56646 139.1975
##  [6,] 53.60809  97.52604 157.4066
##  [7,] 48.45712  85.91990 135.5717
##  [8,] 42.53734  79.39857 126.6035
##  [9,] 46.94647  86.37688 130.3268
## [10,] 55.91946  99.08946 156.3051
## [11,] 30.56754  62.73925 115.0626
## [12,] 46.16716  81.77622 133.9636
## [13,] 47.61360  89.06004 147.3372
## [14,] 58.90354  96.20368 143.9452
## [15,] 51.03205  92.43621 149.3160
## [16,] 54.64381  93.11470 142.4244
## [17,] 47.32641  89.54037 146.5085
## [18,] 56.03696  95.87359 151.9872
## [19,] 52.00245  97.19214 148.1163
## [20,] 37.92732  83.10425 126.7289
```

- Tenemos un problema, la solución que tenía que haber aparecido era la matriz inicial J1. La solución está en realizar la operación [1]-[2]-[3], ya que el signo de los autovectores puede salir cambiado en el proceso de diagonalización de la matriz.

```
X <- (sqrt(eigen(J3)$values[1]))*(eigen(J3)$vectors[,1])%*%(t(eigen(J2)$vectors[,1])) +
-(sqrt(eigen(J3)$values[2]))*(eigen(J3)$vectors[,2])%*%(t(eigen(J2)$vectors[,2])) +
-(sqrt(eigen(J3)$values[3]))*(eigen(J3)$vectors[,3])%*%(t(eigen(J2)$vectors[,3]))
X
```

```
##       [,1] [,2] [,3]
##  [1,]   58  116  178
##  [2,]   45   79  125
##  [3,]   55   98  162
##  [4,]   51   92  141
##  [5,]   51   87  136
##  [6,]   55   99  156
##  [7,]   46   85  137
##  [8,]   45   79  126
##  [9,]   45   80  135
## [10,]   53   98  158
## [11,]   45   74  103
## [12,]   46   85  132
## [13,]   53   93  143
## [14,]   44   90  153
## [15,]   52   94  148
## [16,]   46   89  148
## [17,]   53   92  143
## [18,]   50   96  154
## [19,]   52   91  152
## [20,]   50   76  127
```