

Teaching statistics in a multidisciplinary context: an experimentation in an Italian high school

Enseñanza de Estadística en un contexto multidisciplinar: una experimentación en un instituto italiano

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Abstract

This paper describes an experiment performed at a Vocational School in Palermo during the school year 2010-11. It was designed to teach Statistics at the Secondary level following a multidisciplinary approach. The didactic activity involved the mathematics, Italian, English language and science teachers. We adapted materials and methods from the M@t.abel E-learning program, which is planned for learning basic mathematics in a middle school, in order to use it for experimentation at a professional school. The assessment of the experimentation is based on teachers' evaluation of each student's whole learning process. We compare the learning outcomes of students who participated in the program with others who did not.

Keywords: Statistics education research, multidisciplinary context, experimentation.

Resumen

Este artículo describe un experimento realizado en una Escuela técnica en Palermo durante el año escolar 2010-11. Fue diseñado para enseñar la Estadística en el nivel Secundario con un acercamiento multidisciplinar. La actividad didáctica implicó a los profesores de matemáticas, italiano, inglés y ciencias. Adaptamos materiales y métodos del programa de Enseñanza virtual de M@t.abel, planificado para aprender matemáticas básicas en una escuela secundaria, para usarlo para la experimentación en una escuela profesional. La evaluación de la experimentación está basada en la evaluación de los profesores del proceso de aprendizaje entero de cada estudiante. Comparamos los resultados de los estudiantes que han participado en el programa con los otros que no han participado.

Palabras claves: Investigación de educación estadística, contexto multidisciplinario, experimentación.

1. Introduction

The Italian scientific community has long debated the problems related to teaching Statistics at school. A recently issued reform has increased the role of statistics and probability within the courses of mathematics in all secondary schools¹.

Statistics is taught as a part of the mathematical curriculum like algebra or geometry and in the time frame that is allocated to mathematics (Batanero et al., 2011). According to Usiskin (2014) there are many reasons to do it. First, statistical formulas involve mathematics. Secondly, statistics is in many schools a relatively new in the curriculum, so if statistics is included in mathematics, there is no need for separate time for statistics. Finally, mathematics is tested everywhere and it is taught everywhere. So, if we place statistics within mathematics, we can be more confident that statistics will be tested and taught.

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But, according to Cazorla (2006), Contreras et al. (2011), Peck and Gould (2005), and Ottaviani (2011) there are still gaps in mathematics teachers' training regarding statistical concepts and statistical thinking, and the time available does not allow an increased number of teaching hours (Anichini 2010; Rigatti-Luchini, 2010.). Furthermore, teachers are not aware of the richness of the statistical content, and often think that descriptive statistics is boring. Consequently, they relegate statistics to the end of the school year or leave it out completely because of lack of time (Gattuso, 2008, Gattuso and Ottaviani, 2011).

The Program for International Student Assessment (PISA) has given a strong impulse to the teaching of statistics in the mathematics syllabus since 2000 in Italy (PISA, 2012). In the first years, the PISA assessment, in Italy, yielded negative outcomes in every field, probably because it was aimed at the evaluation of competency and not of knowledge.

In 2010, the Ministry of Education, in the reform of primary and secondary school, has redefined the learning objectives for each curriculum. The Italian National Guidelines introduced Statistics and Probability contents for every school grade, in recognizing the importance for young people to acquire skills that are crucial for the professional and cultural challenges of the new century (Mignani et al., 2014). The Italian reform stresses the importance of multidisciplinary (Giambalvo 2017). A multidisciplinary project involve cultural, historical or philosophical aspects, and teaching a statistical idea as applied mathematics is the first step towards a multidisciplinary approach (Savard & Dominic, 2016). Some of the potentials of multidisciplinary mathematics teaching were discussed in Andresen and Lindenskov (2008). The authors show that multidisciplinary projects increase the students' interest, motivation and engagement in mathematics through the teaching of useful applications of mathematics in authentic, daily life settings. Finally, according to Anderson and Lindenskov, multidisciplinary can be seen as a way to revise the role of school mathematics and, thereby, to insert students' mathematical competence into a broad and reflected view of mathematics and science.

In order to help the multidisciplinary approach to teaching mathematics, the Italian training project: "Mathematics: basic learning with online training" called [M@t.abel](#), has been introduced in 2006. The project consist in training courses for mathematics teachers in middle and secondary schools, and provide modules for classroom experimentation with statistics teaching using prepared materials in a multidisciplinary context. This paper describes an experiment with the statistics teaching modules and materials from [M@t.abel](#), using a competence-based teaching model (Moon, 2008; Zwell, 2000;) in the mathematics curriculum.

2. The Vitruvian man for teaching statistics

After a three-year training course for the teachers (Nicoli, 2011), some teachers in the first year of the handicraft fashion production sector, chose to experiment an interactive and interdisciplinary method for teaching statistics.

Starting from the experience of the learning unit (LU) "The Vitruvian man", presented in the [M@t.abel](#) Project for the third class of a middle school, the teachers planned a competence-based learning unit (LU). The LU was managed by the mathematics teacher, and required a synergy among mathematics, Italian, history, gym, English and

science teachers. This experimentation lasted 50 hours, plus planning time (8 hours) and presentation to the class (2 hours). The class consisted of 22 students.

The experimentation started with the problematic of Leonardo Da Vinci's statements on the Vitruvian man. At the beginning, the students were asked to start from this problem: "Three friends discuss the beauty of the human body. Beauty being a subjective concept, the students wondered if they were any common characteristics to define an admirable body". In the first phase, called "reflections on the problem", the Mathematics teacher led a guided conversation to get the students to reflect on the problem concerning the height, the length of arms and legs, the size of the head compared to the body, and in general the sizes and proportions of the body.

Afterwards, an animated drawing of the Vitruvian man available in the materials of Mat.@bel project was projected, and the teacher asked the following questions: Have you ever seen this image? Where? If you have, do you know who the author is? What makes you think about? How would you describe it to someone who cannot see the image? The students described the image of the Italian one-euro coins specifying that it also represented on the cover of the book "Da Vinci's Code".

In the second step called "Leonardo's statements", the students analyzed some of Leonardo's statements on the Vitruvian man on the ratio between the parts of the human body: "Tanto apre l'omo ne' le braccia, quanto è lla sua alteza", "Dal gomito alla punta della mano fia la quarta parte dell'omo", ("The length of a man's outspread arms is equal to his height", "The distance from the elbow to the tip of the hand is one quarter of a man's height"), etc. Students organized in small groups had recognized that Leonardo's message was to understand the relationships among the various parts of the body.

Meanwhile, the mathematics and Italian teachers used the language of Leonardo (old Italian) for an in-depth study of the historical and linguistic aspects. The Science teacher deepened the concept of man as mirror of the universe and led discussions on the connections between the geometrical figures, the parts of the human body and their physical and spiritual origin. And, the English teacher introduced and reinforced the basic technical language referring to the human body, and the main grammatical categories and invited the students to create brief, simple and coherent texts on the theme using an Italian-English glossary on the main parts of the body.

The third phase concerned the measurement protocol. The Italian and mathematics teachers, working together, asked the students if they thought Leonardo's statements were true. A discussion led the students to recognize the need for experimental verification of Leonardo's hypotheses-statements with measurements. The mathematics teacher told the students they needed to define the population units to be measured and introduced the population and sample concepts and the difference between descriptive and inferential approach. They chose to measure all the pupils of the first classes born in 1996 and they realized that each single student was a statistical unit. The students not born in 1996 prepared the tools to measure the height, the spread of the arms, and the elbow-fingertip length, and they verified the two Leonardo statements: "The length of a man's outspread arms is equal to his height"; "The distance from the elbow to the tip of the hand is one quarter of a man's height".

The data collection phase (4th stage) was done together with the gym teacher, inspired by the lessons on the morphology of the human body. The mathematics teacher guided a conversation on the necessity to round the measurements and the possible mistakes.

This led to a debate about the “approximation” concept, when distinguishing in truncation and rounding approximation (up or down). The data tabulation was done either manually or by spreadsheet (Excel). Measurements were completed on 58 students born in 1996.

In phase 5, “The stem-to-leaf graph”, was introduced to summarize the data collected. After the frequency distributions were constructed, there was a discussion on graphic representations pointing out that the choice of the graph depends on the available data. The teacher introduced the characteristics of the “Stem-and-leaf” graph representing data taking into account the positional value of figures in the decimal system and allowing to save the initial information, passing from a single variable frequency distribution to a class frequency. The students built, first on paper and then on Excel, the stem-and-leaf graph of heights (Fig.1).

14	2
14	
15	1 1 3 4 4 4 4
15	5 5 5 5 6 7 8 8 8 8 9
16	0 0 0 0 0 0 0 0 1 1 2 3 3 3 3 3 4 4
16	5 5 5 6 7
17	0 1 3 3 4
17	6 7 9
18	0 0 0 4
18	5

Figure 1. The stem-and-leaf-graph of students by height

Followed, a presentation on the difference in passing from discrete to continuous valued quantities (phase 6), and the similarity of the stem-and-leaf and a bar graph. Talking about height, the teacher resumed the approximation and the error issues.

During the classroom conversation, the teacher explained that intervals and frequencies could be considered as pairs, and so they could have been drawn on a Cartesian plane as rectangles, whose bases were the widths of the intervals and she introduced the concept of density as a ratio between frequency and width of the interval, in order to determine which heights should have had the rectangles, whose base and surface were known. Then she asked the students to build, both manually and using the spreadsheet, the density distribution of the measured heights and the corresponding graph (Fig.2).

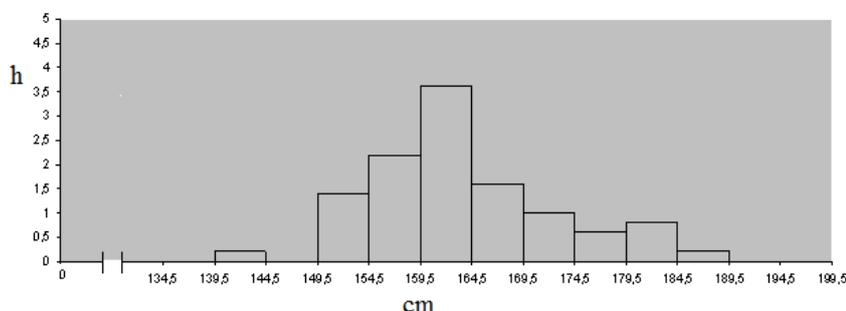


Figure 2. Graphical distribution of students by height

In phase 7, a comparison between the height distribution of male and female students was proposed and some questions were proposed about the maximum and minimum values of male and female students’ heights: “Are there any stems without leaves? What is their meaning? Is it possible to conclude that, even if all the students were born the

same year, as for height they have to be considered differently? How is it possible to establish whether the heights of male students vary more than the heights of female ones?"

To facilitate a graphical comparison, the teacher suggested creating two stem-and-leaf graphs, one referring to the male students and the other to female students (Fig. 3):

Male				Female			
stem 5 cm		leaf cm		stem 5 cm		leaf cm	
14				14	2		
14				14			
15	1	4		15	1	3	4
15	5	5	6	15	5	5	7
16	0	0	0	16	0	0	0
16	5	5	6	16	5		
17	0	1	3	17			
17	6	7	9	17			
18	0	0	0	18			
18	5			18			

Figure 3. The Stem-and-leaf graphs of students by height and gender

Observing and confronting the figure 3, obviously the students concluded that the males were taller than the females!

The teacher specified that in statistical terms, there was variability in both the male and in the female distribution and to measure it, the difference between the maximum and the minimum values in each group, could be obtained. Then, she led the students to recognize that for comparing two distributions of the same variable in different groups, average values could be useful. The concepts of arithmetic mean, of mode, of a variable were introduced and the procedures to compute them were reinforced. The students to, using Excel statistical tools, calculated the mode of heights by gender and, both manually and by spreadsheet, the arithmetic mean of the respective heights. The concept of median and the way to determine it were also introduced. The results are shown in table 1.

Table 1. Summary values referring to heights and arm span by gender in cm

Gender	Statistic units	Mean		Mode		Range Min-Max	
		Height	Arm span	Height	Arm span	Height	Arm span
Male	35	167	167	167	162	34	33
Female	23	159	159	160	156	23	21
Total	58	164	164	160	156	43	47

To verify if Leonardo's hypotheses were acceptable, the question discussed were: "Do the data confirm, on average, Leonardo da Vinci's statements referring to the relations between the parts of the body? Which of the calculated ratios equals to one? What is the value of the arithmetic mean of the ratios?" Students calculated the following ratios: a) arm span/height and b) elbow-fingertip length/height and the mean of these ratios with Excel (Table 2).

The arm span/height ratios were or equal or very close to 1 for ratios for. The same for the elbow-fingertip length/ height ratios, they were or equal or close to 0.25. It also emerged that the mean of arm span/height ratios was equal to 0.99521, that is about 1.00

and that the mean of the elbow-fingertip/height ratio, equal to 0.25181, had a value of about 0.25. The students recognized that Leonardo's statements about the relations among the parts of the human body are still confirmed even after about five hundred years.

Table 2. The arithmetic means of same ratios

Statistic unit	Height	Arm span	Elbow-fingertip length	Arm span /height	Elbow-fingertip length/height
51	155	158	39	1.01935	0.25161
52	185	186	57	1.00541	0.25405
53	165	167	41	1.01212	0.24848
54	156	154	41	0.98718	0.26282
55	173	172	43	0.99422	0.24855
56	163	160	41	0.98160	0.25153
57	180	178	46	0.98889	0.25556
58	174	172	44	0.98851	0.25287
Mean	163	163	41	0.99521	0.25181

The students examined the behaviour of ratios by gender and concluded that the information gathered from the graphs were the same as those contained in tables. Their curiosity was aroused when the teacher asked if there was a relationship between arm span and height and, if so, how would the students verify it empirically. A following discussion had the students see the capacity of the graphical representation to identify a link between two variable and realize the fact that the measures (height, arm span) represented an ordered pair. This led to a representation in a Cartesian plane, with as abscissa $x = \text{heights}$ and as ordinate $y = \text{arm span}$ called a scatter plot.

The students constructed the scatterplot and drew the parallel line to the axes passing through this point. In so doing, the first quadrant of the Cartesian plane was divided into four parts by the parallel lines identifying four new quadrants. They then realized that the dots in the plot were displayed in such a way that with increasing heights, the arm spans increased too. With this recognition, it was possible to state that the dots, altogether, formed a cloud showing an increasing trend and stressed the existence of a direct relationship (Fig. 4).

But the students had more trouble seeing the elbow-fingertip/height relationship. Only a few understood that, if for each pupil the height (x) and the elbow-fingertip (y) were exactly as pointed out by Leonardo, the result would be the line defined by the equation $y = x/4$. They also built the data scatter plot with Excel "scatterplot" including the trend line.

In summary, the students started with the stem-and-leaf graph and the frequency distributions, going on with the average values and the variability indexes and finally to the scatterplot. Going further in-depth, they understood that it would be possible to "find" other statistical techniques and methods to rebut or to affirm Leonardo's several statements. The path had been traced by our active participatory experiment. We could wish the students a pleasant journey in the world of statistics! Buon viaggio!

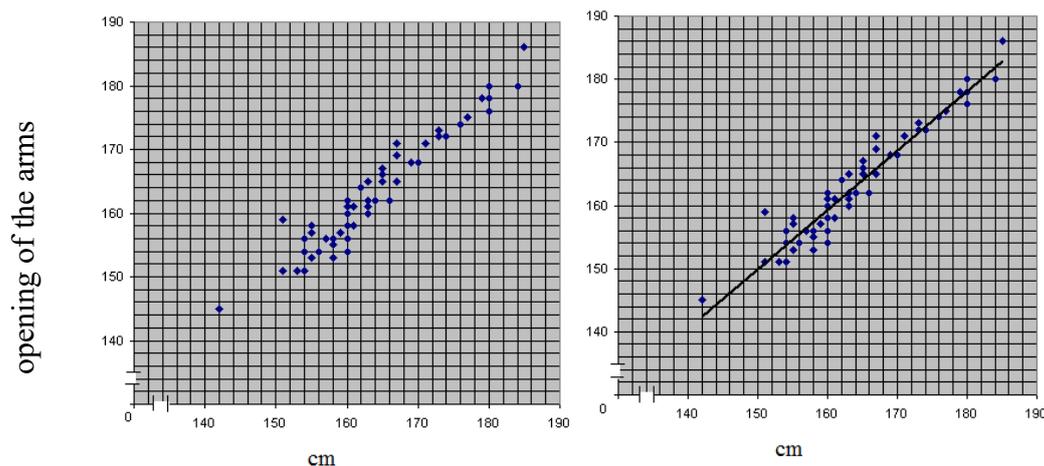


Figure 4. Distribution of students by height and opening of the arms

3. The assessment procedure and some results

In order to evaluate students by authentic criteria (Morlaix, 2009), the assessment involved all the class teachers who participated in the experimentation and a collective judgment was expressed examining the evidence (products, processes, languages, reflections, behaviors...).

After defining the macro-competence of “using the experimental method in its various phases to verify hypotheses”, teachers identified the cultural, professional, citizenship and disciplinary competencies, with reference to the Italian national guidelines.

In particular, for the Italian language (L1) and the foreign language (English, L2), two competencies were identified: understanding and producing texts of various types according to the different communicative goals; knowing and using a specific vocabulary.

In the mathematical area three competencies were chosen: confronting and analyzing geometrical figures identifying invariants and relations; identifying appropriate strategies for the solution of problems; analyzing and interpreting data developing reasoning with the help of graphical representations, using intentionally the computing tools. For the scientific-technological area, the identified competence was: examining, describing and analyzing phenomena.

For the assessment, an evaluation grids of cross-curricular Italian-mathematics and English a cross-curricular test was used. Such test was characterized by questions centered on the statistical concepts faced during the experimentation.

The evaluation grid was divided into eight indicators: autonomy information search and management; data tabulation, representation and elaboration; correctness in the use of techniques, procedures and contents; precision and skill in the use of tools and technologies; (ability to use the Excel software and application of the mathematical tools); use of the language (Italian and English); relationship with the trainers and other adults; quality of the documents used in the presentation; presentation.

For each indicator four levels (Advanced, Intermediate, Basic, Not Achieved) were identified. Two students got the maximum score; seven of them had a slightly lower level and five students did not achieve a sufficient level. The skills in which the highest levels were recorded are: autonomy (7.6 on a scale of 1 to 10), precision and ability in

the use of tools and technologies (7.45) and the relationship with trainers and other adults. The lowest average level was found for the search and management of information (only three students got the highest score).

Finally, each student also had a final mark in mathematics. The results in learning statistics of two groups of students are represented (Table 3): the first is the group of students in the experimentation; the second one is a control group (non-experimental). The results for marks 1 to 10 show that both the median and the mean of the grades in Mathematics of the experimental group are higher than those of control group.

Table 3. Results in mathematics by group

Final marks in Mathematics	Mean	s.d.	Median
Experimental group	6.2	1.5	6
Non-experimental group	4.5	1.6	4.5

We would like to underline that this class group, at difference with “non-experimental” ones, over the years, had the chance to reinforce and deepen statistical topics up to sampling and to probability distributions.

4. Discussion

This paper describes a cross-curricular educational experiment performed to teach Mathematics using some Statistics concepts in a multidisciplinary context. In cooperation with the teachers of Italian language and literature, history, English, sciences and gym, the mathematics teacher led an experiment using competence-based teaching contributed to the construction of competencies relative to cultural, subject, professional and citizenship areas; it also intervened in their learning process, contributing to an increased motivation to learn.

The results deal with two aspects: the first one is attributable to the competence-based teaching; the second is linked to the learning outcomes. From a methodological point of view, a series of educational techniques were used: brainstorming, problem solving, cognitive learning, group and single work, teaching workshops, interactive and frontal lectures. The second result of the experimentation is directly linked to the learning of specific subjects, such as Mathematics using Statistics.

Considering the multidisciplinary context of the experimentation, the proposed activities were functional to develop and/or refine some subject skills different from mathematics and statistics. For what concerns Italian (L1), English (L2) and sciences, the results were also positive

In mathematics, the experimentation allowed the students to consolidate the following abilities: using correctly the concept of approximation; performing elementary geometric constructions using basic measurement tools and/or computer tools; representing on the cartesian plane the linear functions; solving problems implying the use of functions, even in a graphical manner, linked to situations of ordinary life, as a first step towards mathematical modeling; producing and verifying speculations on the basis of some identified regularities; collecting data from measurements; tabulating and representing data by matrices, stem-to-leaf graphs, frequency distributions, histograms (also using the spreadsheet); using in an aware manner absolute, relative and percentage frequencies; choosing, computing and understanding the main measures of center (mean and mode) and variability (range).

More, all the students, even those who stated not to have “a good relationship” with Mathematics, showed a curious and interested attitude towards the experimentation, participated in all the phases and were committed in completing the task. For some students it was a chance to redefine the way to approach Mathematics, and to develop a mature and responsible attitude towards the school experience.

To conclude, in the spirit of learning by doing, starting from the presentation of a problem for which some statements by Leonardo had to be verified and for whose solution quantitative data was necessary, the students faced the main conceptual nuclei of descriptive Statistics. They consolidated and deepened other mathematical knowledge such as the ratio, the rounding, measurement, the cartesian plane and functions. In addition, the essential structures of different types of texts were studied in depth: Leonardo as a scientist and historical figure, the basic technical language in Italian and in English, the parts of the human body in English, the main categories of grammar, and man as mirror of the universe.

Such experimentation can be repeated and it offers the opportunity to consolidate learning starting from Leonardo’s statements. Alas, such experimentations in the Italian school system are difficult to achieve for lack of time and the difficulty to work in team with other teachers. In our case, owing to the very good relationship among the teachers, the biggest obstacle was that of having the students take off their shoes... and sometimes to make measurements, due to the bad odours!

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ⁱ DPR 87/2010, Riordino degli Istituti Professionali; DPR 88/2010, Riordino degli Istituti Tecnici; DPR 89/2010, Riordino dei Licei, cfr. n.3, p.7.