Analysis of models used by student primary teachers when addressing a geometric estimation task

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We present a summary of the results and a sample of illustrative participants' productions of a qualitative research study aimed at exploring the models produced by student primary teachers when addressing an estimation task in which the three spatial dimensions play a major role and the objects whose quantity is to be estimated are deformable. Under a grounded theory approach, we analyse the written productions of 80 student primary teachers who were asked to quantify the amount of bunches of grapes that fit in a box of given dimensions. We highlight the diversity of models (linearisation, base unit, density, direct estimate, counting; also found in related studies) and of magnitudes (volume, length, area, weight) on which their models were based. Geometric considerations (e. g., using a geometric body to model a bunch of grapes) were mainly in the service of metric interests (calculating volumes) and rarely to spatial ones (e. g., arranging the bunches in the box). Our findings extend and add value to those observed in related studies.

Categories of models identified in our analysis

• *Linearisation*. The 3D-question is transformed in three 1D-questions. Length, height and width measurements of the content are assumed and the numbers of times that these respectively fit into the length, height and width measurements of the container (expressed in the same units) are obtained. The product of these numbers is offered as the final answer.

• *Base unit*. The measure of a specific magnitude of the container and the measure of the same magnitude of the content (the "unit") are estimated. Then the first is divided by the second and the quotient of this division is given as final answer.

• *Density*. A subspace of the container is prefixed and the number of objects of content that fit in it is found out. Then this number is multiplied by the number of times that the prefixed subspace fits in the container, giving this product as final answer.

• *Direct estimate*. An estimate of the content that fits in the container is directly given without any intermediary calculation. Some considerations can be included but how (if so) they are taken into account in the given estimate is not made explicit.

• *Counting*. It refers to counting the number of content objects needed to fill the container.

Names of the categories were motivated by the nature of the models they house and the existing ones in related research studies (e. g., Albarracín & Gorgorió, 2014; Ferrando & Albarracín, 2021; Segura, 2022).

On the models identified in our analysis

This works describes models used by student primary teachers when addressing a geometric estimation task consisting on quantifying the number of content units (bunches of grapes) that fit in a given container (dimensioned box). According to the approach to face the task, we identified up to five categories of models in the student primary teachers written productions: linearisation, base unit, density, direct estimate and counting. Subcategories of the first three were described under the consideration of two characteristics: the main magnitude involved and the geometric treatment carried out in the models, namely the geometric body to model the content unit and the arrangement of the content in the container. Regarding the first, we highlight that several models and many of the productions are based on a different magnitude to volume. The most frequently used models fall into the categories of linearisation, base unit based on volume and density. We underline the unexpected consideration of the magnitude weight in the base unit model, based on the estimation of the weight of a content unit and an attempt to convert the measure of capacity of the container into “the measure of weight that it can hold”. Regarding the second characteristic, density focused more on the arrangement of the content (stacking in the box) than on its shape, more often considered in base unit models.

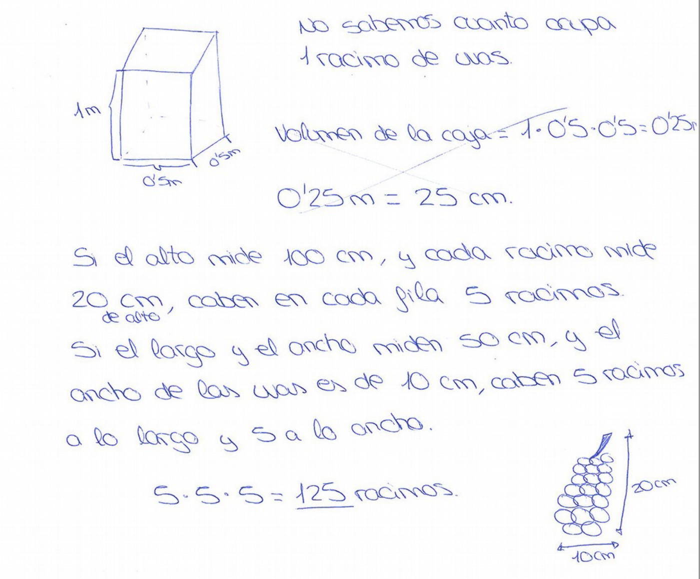
In another order of things, the impact of the geometric features taken into account in the design of the estimation task (namely, the singular shape and deformability of the content and the relevance of the three spatial dimensions) was different in the distinct categories of models. In linearisation, direct estimate and counting they seem to have had less impact. In contrast, base unit models aimed to better geometrically modelling the content unit according to its shape through the use of cones and pyramids for the bunches of grapes, and spheres for the grapes themselves.

On the similarities and differences of the models with those identified in other related studies

Among the main research interests motivating the presented study was to explore if considering deformable or irregularly shaped objects in estimation tasks would lead to new models not based on reductions to 2D estimations as the described in a series of works (e.g., Albarracín & Gorgorió, 2014; Ferrando & Albarracín, 2021; Segura, 2022). Remarkably, the models observed in our study and the categories described on them are closely related to those reported by Segura (2022), being possible to establish a correspondence between both. The impact of the aforementioned features of our proposed task seem to have been on the magnitudes involved in the models and the geometric treatment of the content. In this sense, the categories proposed in our study can be thought as an extension of those described in Segura (2022).

Sample of participants productions illustrating the categories of models

Figures 1-10 show pictures of original participants productions (on the left/up) and their translations to English (on the right/bottom), exemplifying the use of the models described above.



No sabemos cuanto ocupa 1 racimo de uvas.

1m

0'5m

0'5m

~~Volumen de la caja = 1·0'5·0'5 = 0'25m~~

0'25 m = 25 cm.

Si el alto mide 100 cm, y cada racimo mide 20 cm de alto, caben en cada fila 5 racimos.

Si el largo y el ancho miden 50 cm, y el ancho de las uvas es de 10 cm, caben 5 racimos a lo largo y 5 a lo ancho.

5·5·5 = 125 racimos.

20cm

10cm

We do not know how much 1 bunch of grapes occupies

Volume of the box = 1 · 0'5 · 0'5 = 0'25

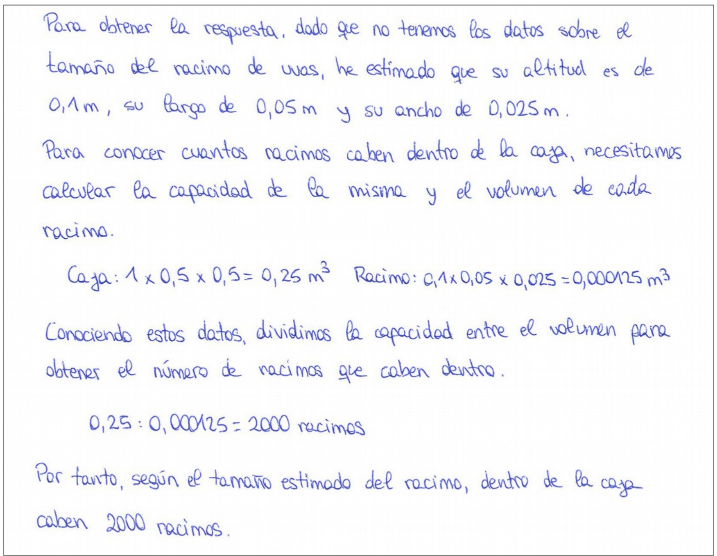
0'25 m = 25 cm

If the height measures 100 cm, and each bunch of grapes measures 20 cm high, 5 bunches fit in each row.

If the length and the width measure 50 cm, and the width of the grapes is 10 cm, 5 bunches will fit in length and width.

5·5·5 = 125 bunches

Figure 1: Linearisation (S158)



Para obtener la respuesta, dado que no tenemos los datos sobre el tamaño del racimo de uvas, he estimado que su altitud es de 0,1 m, su largo de 0,05 m y su ancho de 0,025 m.

Para conocer cuantos racimos caben dentro de la caja, necesitamos calcular la capacidad de la misma y el volumen de cada racimo.

Caja: $1 \times 0,5 \times 0,5 = 0,25\ m^3$ Racimo: $0,1 \times 0,05 \times 0,025 = 0,000125\ m^3$

Conociendo estos datos, dividimos la capacidad entre el volumen para obtener el número de racimos que caben dentro.

0,25 : 0,000125 = 2000 racimos

Por tanto, según el tamaño estimado del racimo, dentro de la caja caben 2000 racimos.

To obtain the response, since we do not have tha data about the size of the bunch of grapes, I have estimated that its altitude is 0'1 m, its length 0'05 m and its width 0,025 cm.

To know how many bunches fit into the box, we need to calculate the capacity of the box and the volume of each bunch.

Box: $1 \times 0,5 \times 0,5 = 0,25\ m^3$

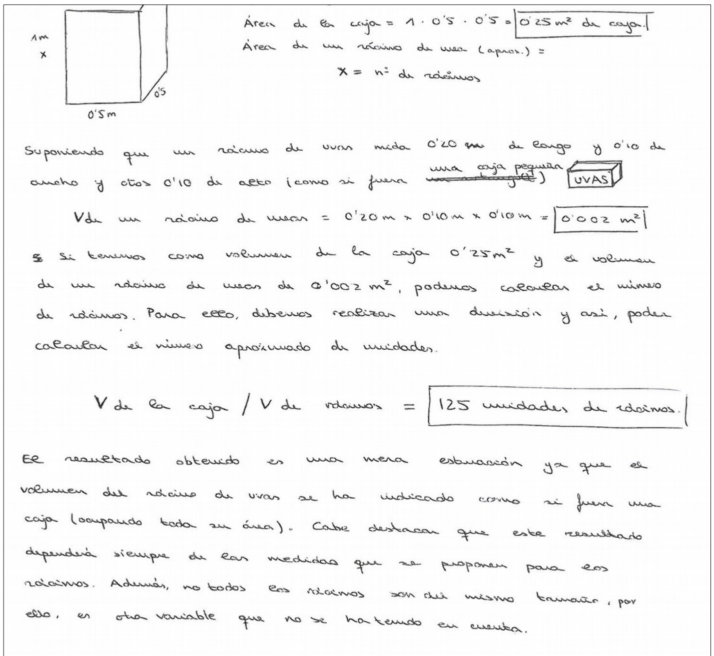
Bunch: $0,1 \times 0,005 \times 0,025 = 0,000125\ m^3$

Knowing these data, we divide the capacity by the voume to obtain the number of bunches that fit inside.

0,25 : 0,000125 = 2000 bunches

Thus, according to the estimated volume of the bunch, 2000 bunches fit in the box.

Figure 2: Base unit based on volume, implicit cuboid, fluid treatment (S76)



1m

0'5m

0'5

Área de la caja = 1 · 0'5 · 0'5 = $\boxed{0'25\ m^2 \text{ de caja.}}$

Área de un racimo de uvas (aprox.) =

X = nº de racimos

Suponiendo que un racimo de uvas mida 0'20 m de largo y 0'10 de ancho y otros 0'10 de alto (como si fuera una caja pequeña) UVAS

V de un racimo de uvas = 0'20 m × 0'10 m × 0'10 m = $\boxed{0'002\ m^2}$

Si tenemos como volumen de la caja 0'25 m^2 y el volumen de un racimo de uvas de 0'002 m^2, podemos calcular el número de racimos. Para ello, debemos realizar una división y así, poder calcular el número aproximado de unidades.

V de la caja / V de racimos = $\boxed{\text{125 unidades de racimos.}}$

El resultado obtenido es una mera estimación ya que el volumen del racimo de uvas se ha indicado como si fuera una caja (ocupando toda su área). Cabe destacar que este resultado dependerá siempre de las medidas que se proponen para los racimos. Además, no todos los racimos son del mismo tamaño, por ello, es otra variable que no se ha tenido en cuenta.

Box area: $1 \times 0.5 \times 0.5 = 0.25\ cm^2$ of box

Area of a bunch of grapes (aprox)

X = number of bunches

Assuming that a bunch of grapes is 0.20 m long and 0.10 widte and 0.10 high (as if it was a small box)

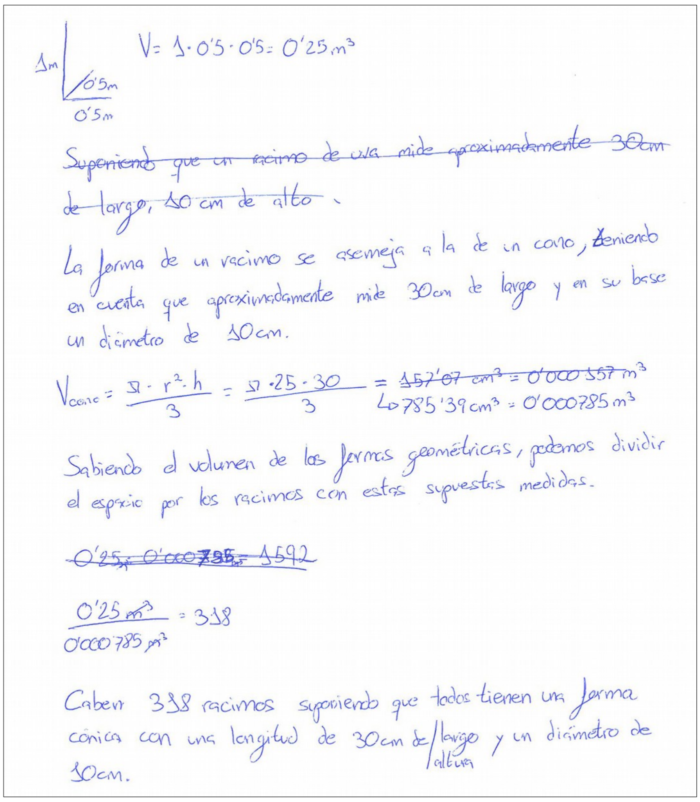
V of a bunch of grapes = $0.20\ m \times 0.10\ m \times 0.10\ m = 0.002\ m^2$

If the volume of the box is 0.25 m^2 and the volume of the bunch of grapes is 0.002 m^2 we can calculate the number of bunches. For this, we must perform a division and hence, we can calculate the approximate number of units.

V of box / V of bunches = 125 units of bunches

The result obtained is a mere estimation, since the volume of the bunches of grapes has been thought as if it was a box (considering all its area). It is worth noting that this result will always depends on the measure of the sizes proposed for the bunches. Moreover, not all bunches are of the same size, this is another variable that has not been taken into account.

Figure 3: Base unit based on volume, explicit cuboid, fluid treatment (S107)



~~Assuming that a cluster of grapes is approximately 30cm long, 10 cm high~~

The shape of a bunch looks like a cone, taking into account that it is approximately 30 cm long and that its base has a diameter of 10 cm.

$V_{cone} = pi \times r^2 \times h / 3 = pi \times 25 \times 30 / 3 =$ ~~$=157.05cm^3=0.000157m^3$~~ $= 785'39\ cm^3 = 0.000785m^3$

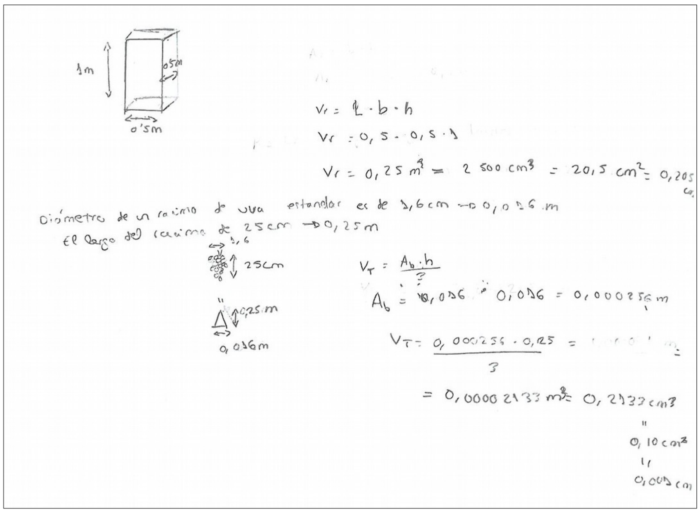
Knowing the volume of the geometric shapes, we can divide the space by the bunches with these assumed sizes.

~~0.25×0.000785=1592~~

$0.25\ \cancel{m^3} / 0.000785\ \cancel{m^3} = 318$

318 bunches fit, assuming that they all have conical shape of 30 cm long / high and a diameter of 10 cm.

Figure 4: Base unit based on volume and cone (S160)

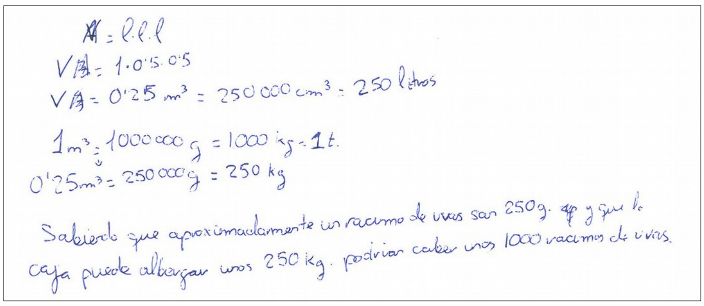


$V_1 = L \times b \times h$
$V_1 = 0,5 \times 0,5 \times 1$
$V_1 = 0,25m^2 = 2500\ cm^3 = 20,5\ cm^2 = 0,205\ cm$

Diameter of standard bunch of grapes is about 1,6 cm → 0,016m
The length of the bunch is 25 cm → 0,25 cm

$V_T = A_b \times h / 3$
$A_b = 0,016 \times 0,016 = 0,000256\ m$
$V_T = 0,000256 \times 0,025/3 =$
$=0,00002133\ m^3 = 0,2133\ cm^3 = 0,10\ cm^2 = 0,001\ cm$

Figure 5: Base unit based on volume and pyramid (S150)

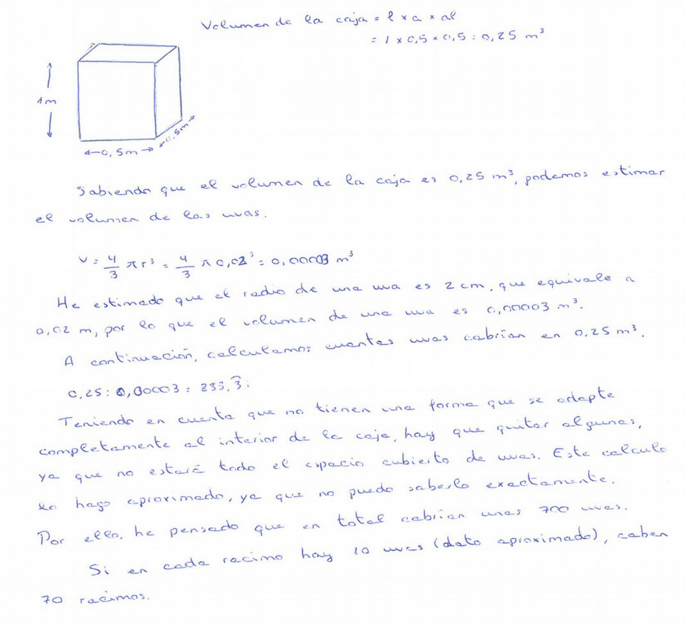


$V = 1 \times 1 \times 1$
$V = 1 \times 0'5 \times 0'5$
$V = 0'25\ m^3 = 250000\ cm^3 = 250$ liters

$1\ m^3 = 1000000\ g = 1000\ kg = 1\ t$
$0'25m^3 = 250000\ g = 250\ kg$

Knowing that a bunch of grapes weighs approximately 250g and that the box can contain about 250 kg, there could fit about 1000 bunches of grapes

Figure 6: Base unit based on weight (S82)


Volumen de la caja = l × a × al
= 1 × 0,5 × 0,5 = 0,25 m^3

1m
0,5m
0,5m

Sabiendo que el volumen de la caja es 0,25 m^3, podemos estimar el volumen de las uvas.

$V = \frac{4}{3} \pi r^3 = \frac{4}{3} \pi 0{,}02^3 = 0{,}00008\ m^3$

He estimado que el radio de una uva es 2 cm, que equivale a 0,02 m, por lo que el volumen de una uva es 0,00003 m^3.
A continuación, calculamos cuantas uvas cabrían en 0,25 m^3.
0,25 : 0,00003 = 255,3.
Teniendo en cuenta que no tienen una forma que se adapte completamente al interior de la caja, hay que quitar algunas, ya que no estará todo el espacio cubierto de uvas. Este calculo lo hago aproximado, ya que no puedo saberlo exactamente.
Por ello, he pensado que en total cabrían unas 700 uvas.
Si en cada racimo hay 10 uvas (dato aproximado), caben 70 racimos.

Volume of the box = l × a × al = 1 × 0,5 ×0,5 = 0,25 m^2

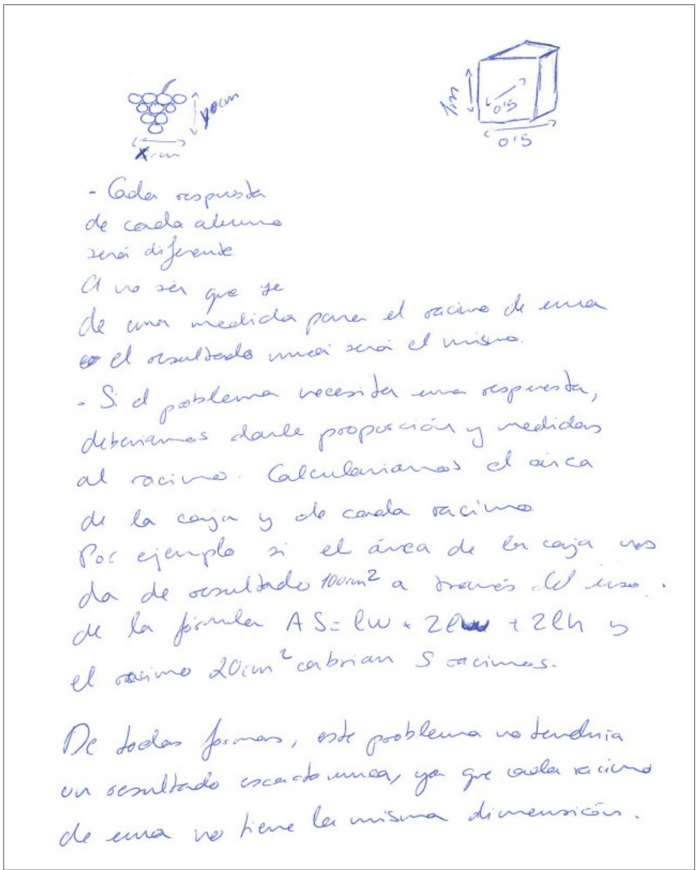
Knowing that the volume of the box is 0,25m^3, we can estimate the volume of the grapes.

V = 4 pi r^3 / 3 = 4 pi $0{,}02^3$ / 3 = 0,00008 m^3

I have estimated that the radius of a grape is 2 cm, which amounts to 0,02 m, so the volume of a grape is 0.00003 m^3.
Next, we calculate how many grapes fit in 0.25 m^3.
0,25 / 0,00003 = 255,3

Taking into account that they do not have a shape that fully adapts to the interior of the box, some ones should be removed, given that not the whole space would be filled by grapes. I perform this calculation approximately, given that I can not know it exactly.
Because of this, I have thought that, in total, about 700 grapes could fit.
If there are about 10 grapes (approximate data) in each bunch, 70 bunches fit.

Figure 7: Base unit based on volume, sphere and non-fluid treatment (S12)


y cm
x cm
1m
0'5
0'5

- Cada respuesta de cada alumno será diferente
A no ser que se de una medida para el racimo de uva el resultado nunca será el mismo.
- Si el problema necesita una respuesta, deberíamos darle proporción y medidas al racimo. Calcularíamos el área de la caja y de cada racimo. Por ejemplo si el área de la caja nos da de resultado 100cm² a través del uso de la fórmula AS = lw + 2lw + 2lh y el racimo 20cm² cabrían 5 racimos.

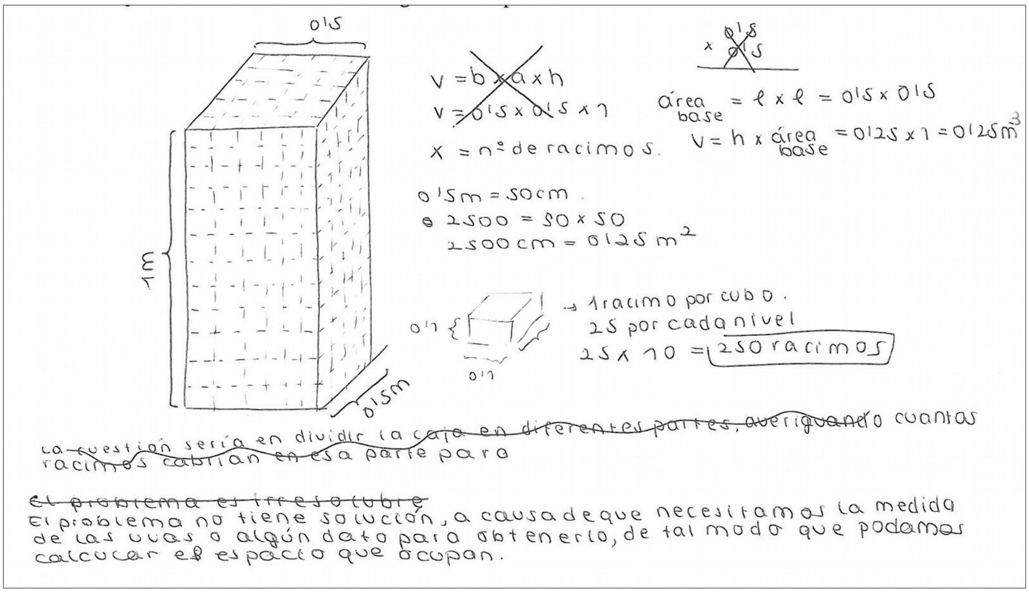
De todas formas, este problema no tendría un resultado exacto nunca, ya que cada racimo de uva no tiene la misma dimensión.

- Each answer of each student will be different.
unless we are given the size of the bunch of grapes, the result will never be the same

- If the problem needs an answer, we should give proportions and sizes to the bunch. We would calculate the area of the box and of each bunch.
For example, if we obtain 100 cm^2 for the area of the box by means of the formula
AS = lw + 2lw + 2lh and the bunch 20cm^2, 5 bunches fit.

Anyways, there would hardly ever be a solution to this problem, since each bunch of grapes is a different size.

Figure 8: Base unit based on area (S10)



~~V=b×a×h~~
~~V=0.5×0.5×1~~ base area = l×l = 0.5 × 0.5
x=number of bunchs V= h × base area= 0.25 × 1 = 0’25 m^3

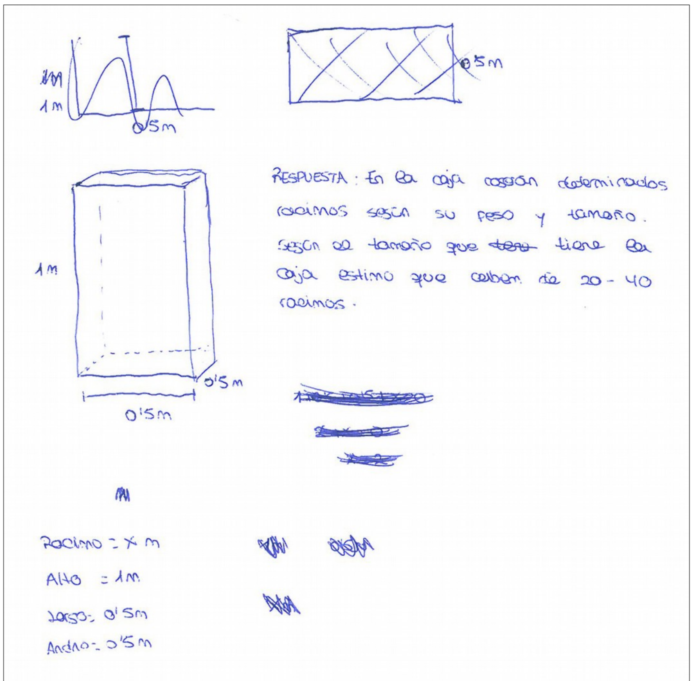
0’5 m = 50 cm
2500 = 50 × 50
2500 cm = 0’25 m^2

1 bunch per cube
25 per level
25 × 10 = 250 bunches

~~The point would be dividing the box in different parts, guessing how many bunches fit in that part for~~

~~The problem is non-solvable~~
The problem has no solutoin, given that we need the size of the grapes or some data to obtain it, so that we can calculate the space that they occupy

Figure 9: Density (S153)



ANSWER: In the box certain bunchs will fit depending on their weight and size. According to the size of the box, I estimate that 20-40 bunches fit.

Bunch = x m
Height = 1 m
Base = 0’5m
Width = 0’5m

Figure 10: Direct estimate (S109)