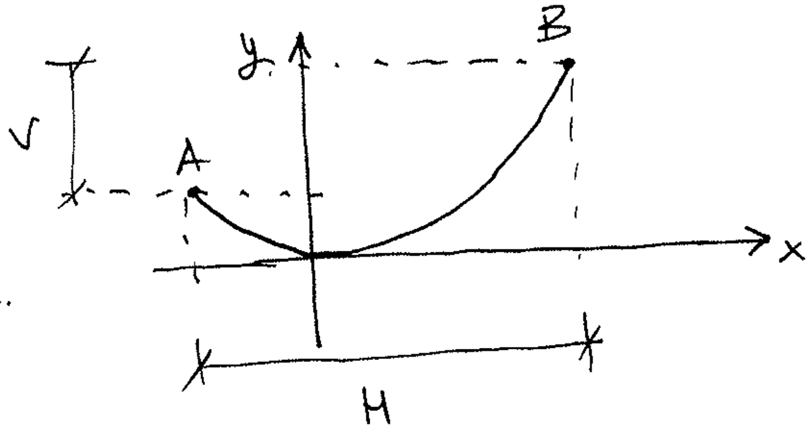
RESOLUCIÓN DE PROBLEMAS DEL CABLE PARABÓLICO

Datos	Incógnitas
γ	x_A, y_A
T_0	x_B, y_B
H	$L.$
V	

$\left.\begin{matrix} x_A, y_A \\ x_B, y_B \end{matrix}\right\}$ situar el vértice.



Ecuaciones:

(1) $x_B = x_A + H$

(2) $y_B = y_A + V$

(3) $y_A = \dfrac{\gamma}{2T_0} x_A^2$.

(4) $y_B = \dfrac{\gamma}{2T_0} x_B^2$.

Sust (1) en (4) $\longrightarrow$ en (2).
(3) en (2).

$$\underbrace{\frac{\gamma}{2T_0}(x_A+H)^2}_{y_B} = \underbrace{\frac{\gamma}{2T_0} x_A^2}_{y_A} + V \;;\; \cancel{\frac{\gamma}{2T_0}}\; \cancel{x_A^2} + H^2 + 2x_A H = \cancel{x_A^2} + \frac{2T_0}{\gamma}\cdot V$$

$$x_A \longrightarrow y_A \qquad x_B \longrightarrow y_B. \qquad \left.\begin{matrix} x_A \\ x_B \end{matrix}\right\} \longrightarrow L$$

$$\boxed{x_A = \frac{1}{2H}\left(\frac{2T_0V}{\gamma} - H^2\right) = \frac{T_0V}{\gamma H} - \frac{H}{2}}$$

$$\boxed{x_B = x_A + H = \frac{T_0V}{\gamma H} + \frac{H}{2}}$$

$$y_A = \frac{\gamma}{2T_0}\left(\frac{T_0V}{\gamma H} - \frac{H}{2}\right)^2 = \frac{\gamma}{2T_0}\left[\left(\frac{T_0V}{\gamma H}\right)^2 + \frac{H^2}{4} - 2\cdot\frac{T_0V}{\gamma \cancel{H}}\cdot\frac{\cancel{H}}{2}\right] =$$

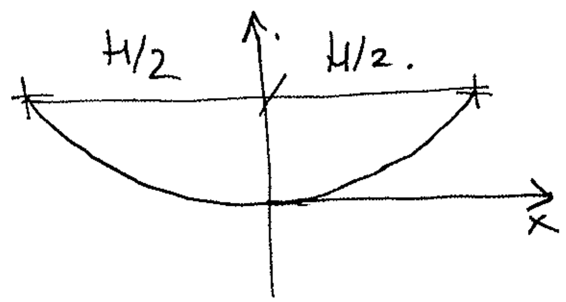
$$= \frac{\cancel{\gamma}}{2\cancel{T_0}}\cdot\frac{T_0^{\cancel{2}}V^2}{\gamma^{\cancel{2}}H^2} + \frac{\gamma}{2T_0}\frac{H^2}{4} - \cancel{2}\,\frac{\cancel{\gamma}}{\cancel{2}\cancel{T_0}}\,\frac{\cancel{T_0}V}{2\cancel{\gamma}} = \boxed{\frac{T_0V^2}{2\gamma H^2} + \frac{\gamma H^2}{8T_0} - \frac{V}{2}}$$

igual $\longrightarrow$ $\boxed{y_B = \dfrac{T_0V^2}{2\gamma H^2} + \dfrac{\gamma H^2}{8T_0} + \dfrac{V}{2}}$

$$L = \int_{x_A}^{x_B} ds = \int_{x_A}^{x_B} \sqrt{1+y'^2}\, dx = \int_{x_A}^{x_B}\left(1 + \frac{\gamma^2}{T_0^2}x\right)^{1/2} dx = \left[\frac{x}{2}\sqrt{1+\frac{\gamma^2}{T_0^2}x} + \frac{T_0}{2\gamma}\,\text{Arcsenh.}\left(\frac{\gamma}{T_0}x\right)\right]_{x_A}^{x_B}$$

Caso particular $V=0$.

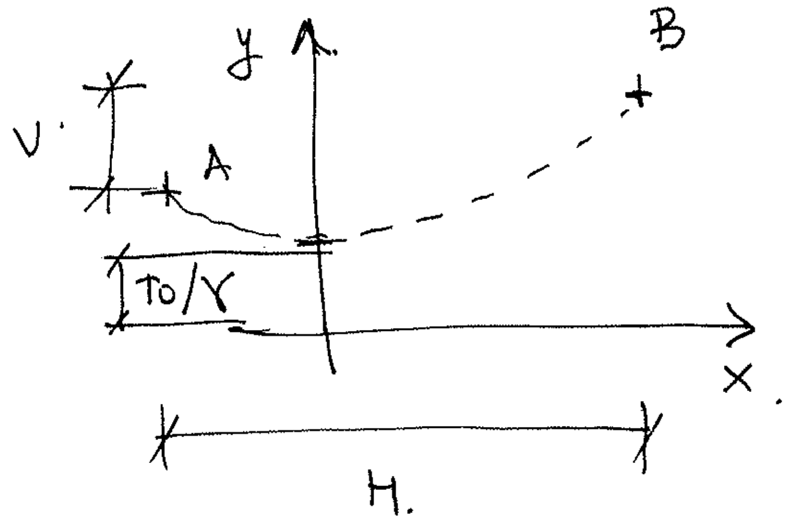
$$x_A = -\frac{H}{2}\;;\quad x_B = \frac{H}{2}\;;\quad y_A = y_B = \frac{\gamma H^2}{8T_0}$$



$$L = \frac{H}{2}\sqrt{1+\frac{\gamma^2H^2}{4T_0}} + \frac{T_0}{\gamma}\,\text{Arcsenh.}\left(\frac{\gamma H}{2T_0}\right)$$

RESOLUCIÓN DE PROBLEMAS DE LA CATENARIA

Datos.	Incógnitas.
γ	x_A, y_A } pto más bajo.
T_0	x_B, y_B
H	L.
V.	



Ecuaciones:

(1) $x_B = x_A + H$.

(2) $y_B = y_A + V$.

(3) $y_A = \dfrac{T_0}{\gamma} \cosh\left(\dfrac{\gamma}{T_0} x_A\right)$.

(4) $y_B = \dfrac{T_0}{\gamma} \cosh\left(\dfrac{\gamma}{T_0} x_B\right)$.

Al igual que en el caso anterior:
sust (1) en (4). $\longrightarrow$ sust en (2).
(3) $\rightarrow$ en (2).

$$\frac{T_0}{\gamma} \cosh\left(\frac{\gamma}{T_0}(x_A + H)\right) = \frac{T_0}{\gamma} \cosh\left(\frac{\gamma}{T_0} x_A\right) + V.$$

resolver numéricamente $\rightarrow x_A \longrightarrow y_A$.
$x_B \rightarrow y_B$.

$$L = \int_{x_A}^{x_B} ds = \int_{x_A}^{x_B} \sqrt{1 + y'^2}\, dx = \int_{x_A}^{x_B} \underbrace{\sqrt{1 + \sinh^2\left(\frac{\gamma}{T_0}x\right)}}_{(ch^2 - sh^2 = 1)}\, dx = \frac{T_0}{\gamma} \sinh \frac{\gamma}{T_0} x \Big]_{x_A}^{x_B}$$

$\sqrt{ch^2} = ch$.

Caso particular $V=0$.

$x_A = -\dfrac{H}{2}$; $x_B = \dfrac{H}{2}$; $y_A = y_B = \dfrac{T_0}{\gamma} \cosh\left(\dfrac{\gamma H}{2T_0}\right)$; $L = \dfrac{2T_0}{\gamma} \sinh\left(\dfrac{\gamma H}{T_0 2}\right)$.