VISCOSITY SUPERSOLUTIONS OF THE EVOLUTIONARY p-LAPLACE EQUATION

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Consider the parabolic p-Laplacian equation

$$\frac{\partial v}{\partial t} = \nabla \cdot \left(|\nabla v|^{p-2} \nabla v\right)$$

We study the regularity of the *viscosity supersolutions* and their spatial gradients. We give a new proof of the existence of ∇v in Sobolev's sense and of the validity of the equation

$$\iint_\Omega \left(-v \frac{\partial \varphi}{\partial t} + \langle |\nabla v|^{p-2} \nabla v,\ \nabla \varphi \rangle \right) dx\ dt \geq 0$$

for all test functions $\varphi \geq 0$. Here Ω is the underlying domain in $\mathbb{R}^{n+1}$ and v is a bounded viscosity supersolution in Ω. The first step of our proof is to establish (1.2) for the so-called infimal convolution v_ϵ. The function v_ϵ has the advantage of being differentiable with respect to all its variables $x_1, x_2, \cdots, x_n$, and t, while the original v is merely lower semicontinuous to begin with. The second step is to pass to the limit as $\epsilon \to 0$. It is clear that $v_\epsilon \to v$ but it is delicate to establish a sufficiently good convergence of the ∇v_ϵ's.

This is joint work with Peter Lindqvist at Trondheim.

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