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Three equations with exponential nonlinearities

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Working for my doctoral thesis, Profs. Brezis and Bénilan asked to investigate the possible existence and properties of the solutions of the semilinear perturbation of the Laplace-Poisson equation

$$\Delta u - e^u + f = 0,$$

posed in $\mathbb{R}^2$ when f is not a mere integrable function but a Dirac mass, or some other measure not given by a only a density. For a simple case put $f(x) = f_1(x) + c\delta(x)$ with f_1 smooth. This was before 1980. I found the now well-known critical value $c_* = 4\pi$: there exist solutions of the problem in the plane with $f_1 \in L^1(\mathbb{R}^2)$, iff $c \leq c_*$. Recently, H. Brezis, M. Marcus, L. Orsina, A. Ponce and others have discussed the general theory elliptic equations with critical values and the theory has now the spirit of functional analysis, which is great.

Around 1993, I. Peral and myself addressed the problem for the evolution version and with the other sign in the exponential (reactive term),

$$\Delta u + \lambda e^u - u_t = 0.$$

The problem was posed in the ball with zero Dirichlet conditions. We linearized the problem around the stationary state, used the Hardy inequality (a novelty then) and proved that stability depends on the space dimension, the critical dimension being 10. The paper was lucky, and many authors have discussed similar results for laplacians and p-laplacians (and more).

10 years passed and then we get another push. In the congress for the 60th anniversary of H. Brezis held in Gaeta I learned of the Paris-Rome activity on the questions about the role of measures in semilinear equations, related to my first paper. But I knew by then that parabolic equations offer some real novelties, like in paper [2], and I had been working on that. My contribution is to see the evolution problem with an exponential in another place and also with a mass. The model is

$$\Delta u - (e^u)_t = 0, \quad u(x,0) = c\delta(x) + f(x),$$

The equation is known in Geometry as the 2-D Ricci flow. The result I will present is surprising and is going to appear in 2007 in DCDS. I am sure Ireneo will like it. Let me end with a question: any idea about what is next?

References:

1. J. L. Vázquez, *On a semilinear equation in $\mathbb{R}^2$ involving bounded measures.* Proc. Royal Soc. Edinburgh 95 A (1983), 181-202.

2. I. Peral, J. L. Vázquez, *Stability and instability of singular solutions of the semilinear heat equation with exponential reaction.* Archive Rat. Mech. Anal., 129, 3 (1995), 201-224.

3. J. L. Vázquez, *Evolution of point masses by planar logarithmic diffusion*, Discrete and Continuous Dyn. Systems, to appear in 2007.