"A stable branch of solutions of a nonlinear Schrödinger equation"

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Abstract

In joint work with François Genoud, we consider the problem

$$\Delta u(x) + V(x)\left|u(x)\right|^{p-1} u(x) - \lambda u(x) = 0 \text{ with } u \in H^1(\mathbb{R}^N)\backslash\{0\} \qquad (1)$$

where $N \geq 3$ and $V \in C^1(\mathbb{R}^N\backslash\{0\})$ is such that

(i) for some $b \in (0,2)$, $\left|x\right|^b V(x)$ is bounded as $x \to 0$

and $\left|x\right|^b V(x) \to 1$ as $|x| \to \infty$,

(ii) $1 < p < 1 + \frac{2(2-b)}{N-2}$.

We prove that there exist $\lambda_0 > 0$ and $U \in C^1((0,\lambda_0), H^1(\mathbb{R}^N))$ such that $(\lambda, U(\lambda))$ satisfies (1) for all $\lambda \in (0,\lambda_0)$. Furthermore, if $1 < p < 1 + \frac{2(2-b)}{N}$, the standing wave $e^{i\lambda t}U(\lambda)(x)$ is an orbitally stable periodic solution of the nonlinear Schrödinger equation

$$i\partial_t w + \Delta w + V(x)\left|w\right|^{p-1} w = 0.$$

We discuss also similar conclusions obtained recently by de Bouard-Fukuizumi and Jeanjean-Le Coz.