Semiclassical States for Nonlinear Schrödinger Equations with Potentials Vanishing at Infinity

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We study the existence and behavior of solutions for the nonlinear Schrödinger equation:

$$-\varepsilon^2 \Delta u + V(x) u = u^p, \quad x \in \mathbb{R}^n, \quad u > 0. \tag{1}$$

for ε small. Here, $1 < p < \frac{n+2}{n-2}$ and $V(x)$ a positive potential, $V(x) \sim |x|^{-\alpha}$ ($|x| \to +\infty$) for $0 \leq \alpha \leq 2$. We show that for ε small enough, there exist bound states concentrating around some point x_0, provided that x_0 is a stable stationary point of $V(x)$.

When the potential $V(x)$ is radial, we find also radial solutions that concentrate around a sphere. The radius of that sphere is given by the critical points of a certain auxiliary weighted potential.

In both cases, the limit exponent $\alpha = 2$ seems to be a limiting exponent in order to find solutions with finite energy.

The proofs use a perturbation technique in a variational setting, through a Lyapunov-Schmidt reduction.