HOW TO APPROXIMATE THE HEAT EQUATION WITH NEUMANN BOUNDARY CONDITIONS BY NONLOCAL DIFFUSION PROBLEMS

1. Abstract

The purpose of this talk is to show that the solutions of the usual Neumann boundary value problem for the heat equation can be approximated by solutions of a sequence of nonlocal "Neumann" boundary value problems.

Let $J : \mathbb{R}^N \to \mathbb{R}$ be a nonnegative, radial, continuous function with $\int_{\mathbb{R}^N} J(z)\,dz = 1$. Assume also that J is strictly positive in $B(0,d)$ and vanishes in $\mathbb{R}^N \setminus B(0,d)$. Nonlocal evolution equations of the form $u_t(x,t) = (J*u-u)(x,t) = \int_{\mathbb{R}^N} J(x-y)u(y,t)\,dy - u(x,t)$, and variations of it, have been recently widely used to model diffusion processes, see [3]. In this talk, following [1] and [2], we propose a nonlocal "Neumann" boundary value problem, namely

$$u_t(x,t) = \int_\Omega J(x-y)\big(u(y,t)-u(x,t)\big)\,dy + \int_{\mathbb{R}^N\setminus\Omega} G(x,x-y)g(y,t)\,dy, \tag{1.1}$$

where $G(x,\xi)$ is smooth and compactly supported in ξ uniformly in x. Now, for given J and G we consider the rescaled kernels $J_\varepsilon(\xi) = C_1 \frac{1}{\varepsilon^N} J\left(\frac{\xi}{\varepsilon}\right)$, $G_\varepsilon(x,\xi) = C_1 \frac{1}{\varepsilon^N} G\left(x, \frac{\xi}{\varepsilon}\right)$ and then the solution $u^\varepsilon(x,t)$ to (1.1) with J_ε and G_ε verifies

$$u^\varepsilon \to u,$$

in different topologies according to different choices of the kernel G. Here u is the solution of the heat equation, $u_t = \Delta u$ with boundary condition $\partial u/\partial\eta = g$ and initial condition u_0.

This is a joint work with C. Cortazar, M. Elgueta and N. Wolanski.

References

[1] C. Cortazar, M. Elgueta, J. D. Rossi and N. Wolanski. *Boundary fluxes for non-local diffusion.* To appear in J. Differential Equations.

[2] C. Cortazar, M. Elgueta, J. D. Rossi and N. Wolanski. *How to approximate the heat equation with Neumann boundary conditions by nonlocal diffusion problems.* To appear in Arch. Rat. Mech. Anal.

[3] P. Fife. *Some nonclassical trends in parabolic and parabolic-like evolutions.* Trends in nonlinear analysis, 153–191, Springer, Berlin, 2003.

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