A NEW APPROACH FOR NONLINEAR ELLIPTIC EQUATIONS WITH L^1 DATA

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ABSTRACT. I will present some results contained in the joint paper with A. Alvino [2]. We consider a class of nonlinear elliptic problems whose prototype is

$$-\mathrm{div}(|\nabla u|^{p-2}\nabla u) = f \quad \text{in } \Omega, \qquad u = 0 \quad \text{on } \partial\Omega, \tag{0.1}$$

where Ω is a bounded open subset of $\mathbb{R}^N$, $N \geq 2$, $p > 1$ and the datum f is a function belonging to $L^1(\Omega)$. A natural way to introduce a notion of solution to (0.1) is to proceed by an approximation method as in [3]. The idea consists in fixing the solution as the limit of a sequence of solutions to (0.1) which, owing to the regularity of the right-hand side, belong to the energy space $W_0^{1,p}(\Omega)$. In [2] we prove existence and uniqueness for such a solution by using a different and simpler approach relying on symmetrization methods. In this way we stress that our investigation could be confined to problems related to the p-Laplace operator with spherically symmetric data. We use suitable test functions in order to prove both apriori estimates and continuity dependence on the data; finally we deduce in a natural way both existence and uniqueness results. This approch use estimates and comparison results contained in [1] and [4].

[1] A. ALVINO, Boll. Un. Mat. Ital. 14 (1977), 148–156.
[2] A. ALVINO, A. MERCALDO, Preprint.
[3] L. BOCCARDO, T. GALLOÜET, J. Funct. Anal. 87 (1989), 149–169.
[4] G. TALENTI, Boll. Un. Mat. Ital. 4 (1985), 917–949.