The Neumann Problem for Nonlocal Nonlinear Diffusion Equations

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We are interested in nonlocal diffusion models of the form

$$P_\gamma^J(z_0) \quad \begin{cases} z_t(t,x) = \displaystyle\int_\Omega J(x-y)(u(t,y)-u(t,x))\,dy, & x \in \Omega,\ t>0, \\ z(t,x) \in \gamma(u(t,x)), & x \in \Omega,\ t>0, \\ z(0,x) = z_0(x), & x \in \Omega. \end{cases} \tag{1}$$

Here Ω is a bounded domain, $z_0 \in L^1(\Omega)$, γ is a maximal monotone graph in $\mathbb{R}^2$ such that $0 \in \gamma(0)$, and $J : \mathbb{R}^N \to \mathbb{R}$ is a nonnegative continuous radial function with $\int_{\mathbb{R}^N} J(r)dr = 1$ and $0 \in \mathrm{int}[\mathrm{supp}(J)]$.

This is a nonlocal diffusion problem analogous with the usual Laplacian with Neumann boundary conditions. We prove existence and uniqueness of solutions with initial conditions in $L^1(\Omega)$. Moreover, when γ is a continuous function we find the asymptotic behaviour of the solutions, they converge as $t \to \infty$ to the mean value of the initial condition.