Singular solutions of the Gelfand problem in perturbations of the ball

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We consider the Gelfand problem:

$$\begin{cases} -\Delta u = \lambda e^u & \text{in } \Omega \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

where $\Omega \subset \mathbb{R}^N$ is a smooth bounded domain and $\lambda > 0$. In any dimension $N \geq 3$ and for Ω the unit ball the function $-2\log|x|$ is a solution for $\lambda = 2(N-2)$. In dimension $N \geq 4$ we show that if Ω is close enough to a ball in an appropriate sense there is a singular solution (λ, u) with $u \sim -2\log|x-\xi|$ for some point $\xi \in \Omega$ and $\lambda \sim 2(N-2)$.

In general domains it is known that solutions exist for $0 \leq \lambda \leq \lambda^*$ where $\lambda^* > 0$ and is finite, and that the problem has a unique, possibly singular, solution when $\lambda = \lambda^*$. When Ω is the unit ball it is known that if $N \leq 9$ then u^* is a classical solution, and if $N \geq 10$ then $u^* = -2\log|x|$. We show that if Ω is close enough to a ball and $N \geq 11$ then u^* is singular with $u^* \sim -2\log|x-\xi|$ for some point $\xi \in \Omega$.

This is joint work with Louis Dupaigne (LAMFA, Université Picardie Jules Verne).