On the attainability of the optimal constants of some Caffarelli-Kohn-Nirenberg inequalities with mixed boundary conditions. Applications.

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Abstract. In this talk, will be presented some results on the optimal constants of Sobolev and Hardy-Sobolev inequalities with weights and their relation with the behavior of some mixed Dirichlet-Neumann boundary conditions. Will be analyzed the attainability of the Sobolev constant

$$S^2_\gamma(\Omega,\Sigma_1)=\inf_{u\in E^{2,\gamma}_{\Sigma_1}(\Omega);u\not\equiv 0}\frac{\displaystyle\int_\Omega |x|^{-2\gamma}|\nabla u|^2dx}{\Big(\displaystyle\int_\Omega |x|^{-2^*\gamma}|u|^{2^*}dx\Big)^{\frac{2}{2^*}}}, \tag{1}$$

and the Hardy-Sobolev constant

$$\Lambda_{N,\gamma}(\Omega,\Sigma_1)=\inf_{u\in E^{2,\gamma}_{\Sigma_1}(\Omega),u\not\equiv 0}\frac{\displaystyle\int_\Omega |x|^{-2\gamma}|\nabla u|^2dx}{\displaystyle\int_\Omega \frac{|u|^2}{|x|^{2(\gamma+1)}}dx} \tag{2}$$

where $\Omega\subset\mathbb{R}^{\mathrm{N}}$, $N\geq 3$, is a smooth bounded domain such that $0\in\Omega$, $-\infty<\gamma<\frac{N-2}{2}$, $2^*=\frac{2N}{N-2}$, and $E^{2,\gamma}_{\Sigma_1}(\Omega)$ the natural Sobolev space in this framework. The deep relation between the geometry of the domain, the boundary conditions and the attainability of the critical constants will be showed. As a direct consequence, some applications to elliptic problems like

$$\left\{\begin{array}{rcl} -\mathrm{div}(|x|^{-2\gamma}\nabla u) &=& \lambda\dfrac{u^q}{|x|^{2(\gamma+1)}}+\dfrac{u^r}{|x|^{(r+1)\gamma}},\quad u>0\quad \text{in }\Omega,\\ u &=& 0\quad \text{on }\Sigma_1,\\ |x|^{-2\gamma}\dfrac{\partial u}{\partial\nu} &=& 0\quad \text{on }\Sigma_2,\end{array}\right. \tag{3}$$

will be discussed, where q and r are given real parameters under convenient hypotheses and $\overline{\Sigma}_1$, $\overline{\Sigma}_2$, provide a smooth partition of $\partial\Omega$.

The main results are contained in the following paper:

B. Abdellauoi, E. Colorado, I. Peral, *Effect of the boundary conditions in the behavior of the optimal constant of some Caffarelli-Kohn-Nirenberg inequalities. Application to some doubly critical nonlinear elliptic problems.* Adv. Differential Equations **11** (2006), no. 6, 667-720.