Reaction-diffusion elliptic equations and minimal surfaces

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We will present recent developments on solutions of reaction-diffusion elliptic equations that are strongly related to some classical results in the theory of minimal surfaces. A classical result in minimal surface theory is the flatness of minimal graphs up to dimension 7. Its semilinear analogue is a conjecture posed by E. De Giorgi in 1978 for which progress has been made only recently. We will describe some of these developments. They concern the stability, the minimality, the monotonicity in one variable, and the symmetry of different types of solutions: monotone solutions, minimizers, radial solutions, and saddle-shaped solutions.