

Elliptic Problems with quasilinear term: The effect of the gradient term on the existence result.

Boumediene Abdellaoui*

In this work we analyze existence, nonexistence, multiplicity and regularity of solution to problem

$$\begin{cases} -\Delta u \pm |\nabla u|^p = \lambda \frac{u}{|x|^2} + \alpha f(x) & \text{in } \Omega \\ u > 0 & \text{on } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases} \quad (1)$$

where Ω is a bounded open set in \mathbb{R}^N such that $0 \in \Omega$, $p > 1$, $\alpha > 0$ and $f(x)$ is a positive measurable function under some hypothesis that we will precise later.

Elliptic equations with gradient term have been widely studied in the literature.

The main goal of this work is to get the main interaction between the gradient term and the hardy potential to get existence, no-existence and multiplicity results for problem (1).

Our work is divided in two main cases:

The absorption effect:

If the gradient term $|\nabla u|^q$ appears in (1) as an absorption term, then we show the existence of solution for all $\lambda > 0$ and for all $f \in L^1(\Omega)$, moreover the presence of $|\nabla u|^p$ is sufficient to break down any resonance effect of the linear term.

The reaction effect:

If the gradient term $|\nabla u|^q$ appears in (1) as a reaction term then we have two main cases:

1. If $p \geq p_+(\lambda)$, to be given, then there is no solution in a very weak sense.
2. If $p < p_+(\lambda)$, with some additional hypotheses on f and α , there exists an entropy solution, moreover under some extra conditions on Ω and f , we get the existence of infinity many solutions.

Referencias

- [1] B. Abdellaoui, A. Dall'Aglio, I. Peral, *Some Remarks on Elliptic Problems with Critical Growth in the Gradient*. J. Differential Equations, **222**, (2006), 21-62
- [2] B. Abdellaoui-I. Peral, *Viscous Hamilton-Jacobi Equations with a Hardy Potential: The Optimal Power*, to appear.

*Département de Mathématiques, université de Tlemcen, Algeria.

- [3] B. Abdellaoui, I. Peral, A. Primo, *Some elliptic problems with Hardy potential and critical growth in the gradient: non-resonance and blow-up results*, To appear.
- [4] N. Alaa, M. Pierre, *Weak solutions of some quasilinear elliptic equations with data measures*. SIAM. J. Math. Anal. **24**, no. 1 (1993), 23-35.
- [5] L. Boccardo, F. Murat, J.P. Puel, *Existence de solutions faibles pour des équations elliptiques quasi-linéaires à croissance quadratique*, in Nonlinear Partial Differential Equations and their Applications, Collège de France Seminar, Vol. IV (J.-L. Lions and H. Brezis, eds.), Research Notes in Math, **84**, Pitman, London, 1983, 19-73.
- [6] V. Ferone, B. Messano, *Comparison and existence results for classes of nonlinear elliptic equations with general growth in the gradient*. To appear.