The Schur Siegel trace problem

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Abstract

Let α be an algebraic integer of degree d with minimal polynomial P, that is, $P(x) = x^d + a_1 x^{d-1} + \cdots + a_d$, $a_k \in \mathbb{Z}$, $P(\alpha) = 0$, and any other polynomial vanishing on α is a multiple of P. Then P has d different roots, $\alpha_1, \ldots, \alpha_d$. If all of them are positive, then α is said to be totally positive. Let

$$\mathcal{T} = \left\{ \frac{1}{d} \sum_{k=1}^{d} \alpha_k : \alpha \text{ is a totally positive algebraic integer} \right\}.$$

The Schur-Siegel trace problem asks wether $[\,1, 2 - \varepsilon) \cap \mathcal{T}$ is finite for all $\varepsilon > 0$.

We will present new results on the problem, obtained in collaboration with J. C. Peral, and its relation to the integer Chebyshev problem.